G10 TERM 1 TEST 1

Wed 14 Nov 2018

1)

1) $1 \cdot 2^{10} + 1 \cdot 2^{7} + 1 \cdot 2^{1} = 1024 + 128 + 2 = 1154$

2) $1 \cdot 2^{11} + 1 \cdot 2^{4} + 1 \cdot 2^{2} = 2048 + 16 + 4 = 2068$

3) $1 \cdot 2^{9} + 1 \cdot 2^{6} + 1 \cdot 2^{3} = 512 + 64 + 8 = 584$

4) $1 \cdot 2^{8} + 1 \cdot 2^{5} + 1 \cdot 2^{0} = 256 + 32 + 1 = 289$

5) $2^{4} - 1 = 16 - 1 = 15$

6) $2^{8} - 1 = 256 - 1 = 255$

7) $2^{12} - 1 = 2^{2} \cdot 2^{10} - 1 = 4096 - 1 = 4095$

8) $2^{11} - 1 = 2048 - 1 = 2047 = \left\lfloor \frac{4095}{2} \right\rfloor$

2) 011 $\longrightarrow$ 001 It corresponds to $\underbrace{\text{integer division}}$ by 2

3 $\frac{3}{2}=1.5 \longrightarrow$ 1

The smallest integer closest to the division by 2

Example: 00010000 = 16

00001000 = 8 $\frac{16}{2}=8$

00001111 = 15

00000111 = 7 $\frac{15}{2}=7.5$

3) We'll do 362 & 127 by division method & the rest we'll guess

1) $251 = 255 - 4$

$255 = 1111\ 1111$

$251 = 1111\ 1011$

2) $66 = 64 + 2$

$64 = 0100\ 0000$

$66 = 0100\ 0010$

3)

	int. div	remainder
127/2	63	1
63/2	31	1
31/2	15	1
15/2	7	1
7/2	3	1
3/2	1	1
1/2	0	1

Hence $127 = 0111\ 1111$

4)

362/2	181	0
181/2	90	1
90/2	40	0
45/2	22	1
22/2	11	0
11/2	5	1
5/2	2	1
2/2	1	0
1/2	0	1

Hence $362 = 1\ 0110\ 1010$

4) FIRST LONG WAY

1) $00F = 15$

2) $0FF = F \cdot 16^1 + F \cdot 16^0 = 15 \cdot 16 + 15 \cdot 1 = 240 + 15 = 255$

$15 \cdot 16 = (16-1)16 = 16^2 - 16 = 256 - 16 = 240$

3) $FFF = F \cdot 16^2 + \underbrace{F \cdot 16^1 + F \cdot 16^0}_{255} = 15 \cdot 256 + 255 = 3840 + 255 = 4095$

$15 \cdot 16^2 = (16-1) \cdot 16^2 = 16^3 - 16^2$

$15 \cdot 16^2 = 4096 - 256$

$= 3840$

$16^3 = 4096$

$16^2 = 256$

4) $FF0 = F \cdot 16^2 + F \cdot 16^1 + 0 = 15 \cdot 16^2 + 15 \cdot 16 = 3840 + 240 = 4080$

4) Short(er) way

1) $00F = 15$

2) $0FF = 100 - 1 = 16^2 - 1 = 256 - 1 = 255$

3) $FFF = 1000 - 1 = 16^3 - 1 = 4096 - 1 = 4095$

4) $FF0 = FFF - F = 4095 - 15 = 4080$

5) In hexadecimal, shifting all digits 1 position to the left means multiplying by 16

$00F = 15$

$0F0 = 15 \cdot 16 = 240$

6)

1)

	Int. division by 16	Remainder
251/16	15	11
15/16	0	15

$16 \cdot 15 = 240$

$\boxed{\text{Hence } 251 = FB}$

2)

66/16	4	2
4/16	0	4

$\boxed{\text{Hence } 66 = 42 \text{ in hexadecimal}}$

3)

177/16	7	15
7/16	0	7

$7 \cdot 16 = 70 + 42 = 112$

Hence

$\boxed{177_{10} = 7F}$

4)

362/16	22	10
22/16	1	6
1/16	0	1

$$\begin{array}{r} 362 \\ -32 \\ \hline 42 \\ -32 \\ \hline 10 \end{array} \quad 362 \div 16 = 22$$

Hence

$\boxed{362 = 16A}$

6) Converting from to binary

1) $251 = 255 - 4 = 1111\ 1111 - 0000\ 0100 = 1111\ 1011 = \boxed{= F\ B}$

2) $66 = 64 + 2 = 0100\ 0010 = \boxed{= 0x42}$

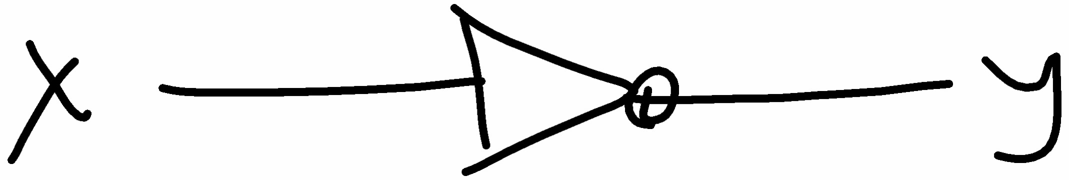
3) $127 = 128 - 1 = 1000\ 0000 - 1 = 0111\ 1111 = \boxed{= 7F}$

4) $362 = 256 + 106 =$

$= 256 + \underbrace{64 + 32 + 10}_{0110\ 1010} = 1\ 0000\ 0000 + 0110\ 1010 =$

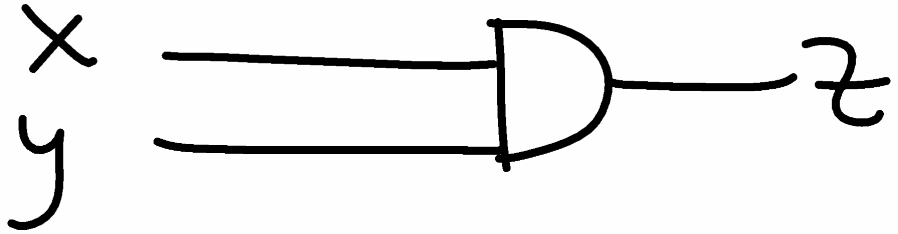
$= 1\ 0110\ 1010 = \boxed{= 16A}$

7) NOT



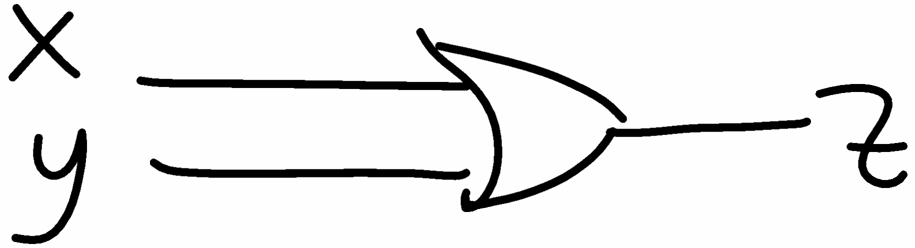
x	y
0	1
1	0

AND



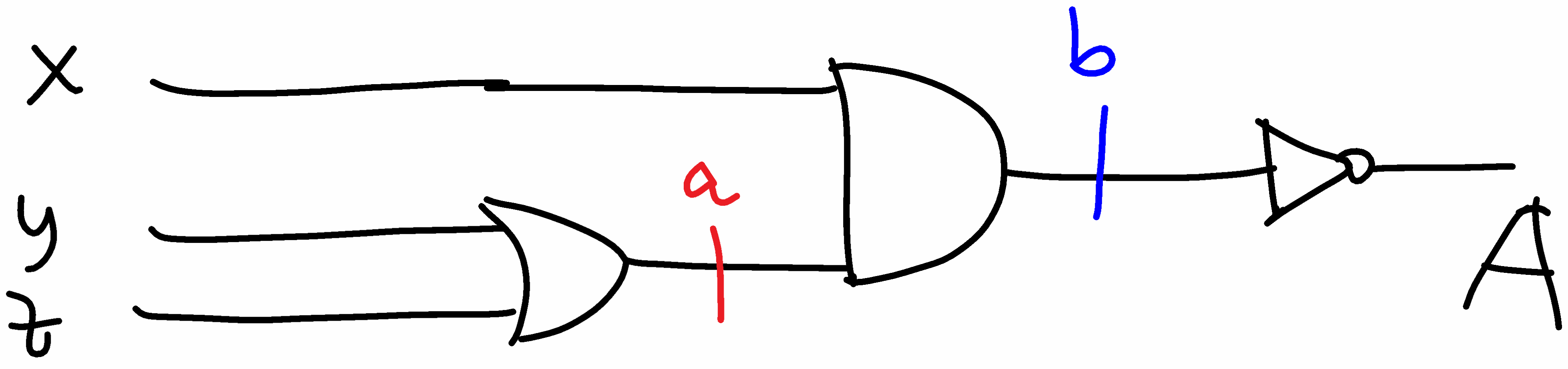
x	y	z
0	0	0
0	1	0
1	0	0
1	1	1

OR



x	y	z
0	0	0
0	1	1
1	0	1
1	1	1

8) NOT (x AND (y OR z))



x	y	z	A
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0

Rows 1–4: As x=0 it is b=0. Hence A=1

Row 5: → a=0 ⇒ b=0

Rows 6–8: Both a=1 & x=1 Hence b=1 & A=0

2) NOT (x AND y)

x	y	NOT (x AND y)
0	0	1
0	1	1
1	0	1
1	1	0

(NOT x) OR (NOT y)

x	y	(NOT x) OR (NOT y)
0	0	1
0	1	1
1	0	1
1	1	0

Both expressions have the same truth tables

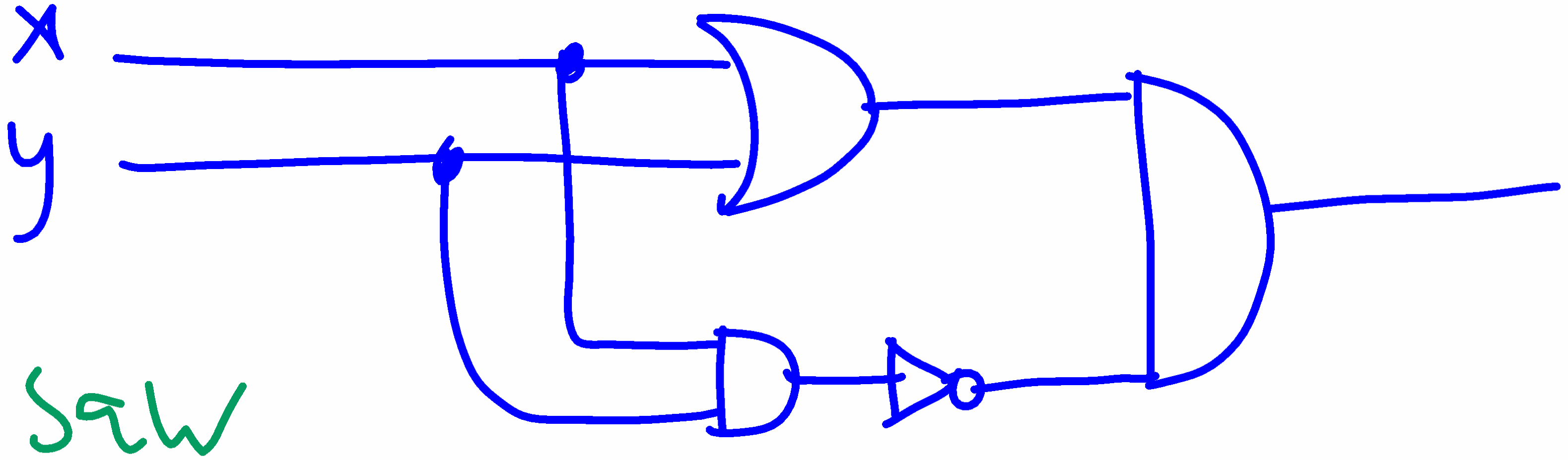
Hence, both expressions are equivalent.

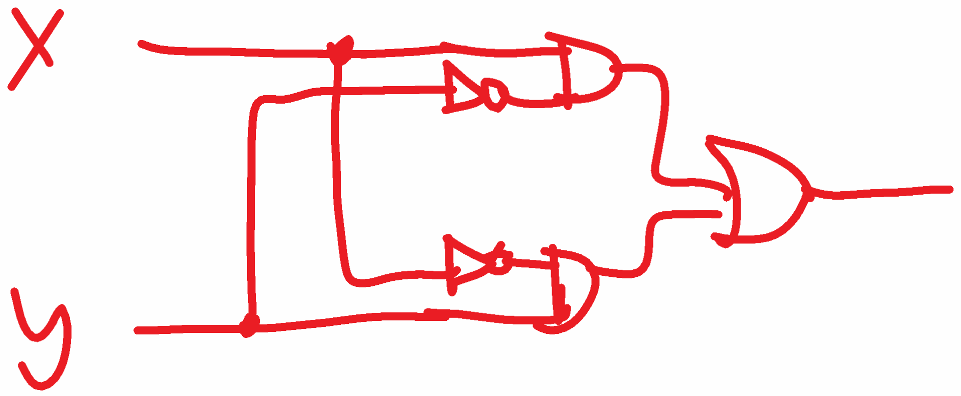
10) $x \text{ XOR } y = (x \text{ OR } y) \text{ AND } (\text{NOT } (x \text{ AND } y))$ (A)

$= (x \text{ AND } (\text{NOT } y)) \text{ OR } ((\text{NOT } x) \text{ AND } y)$ (B)

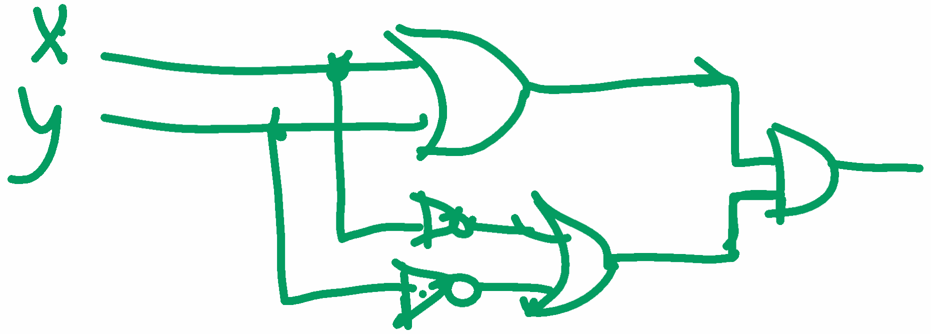
The diagrams are

(A)

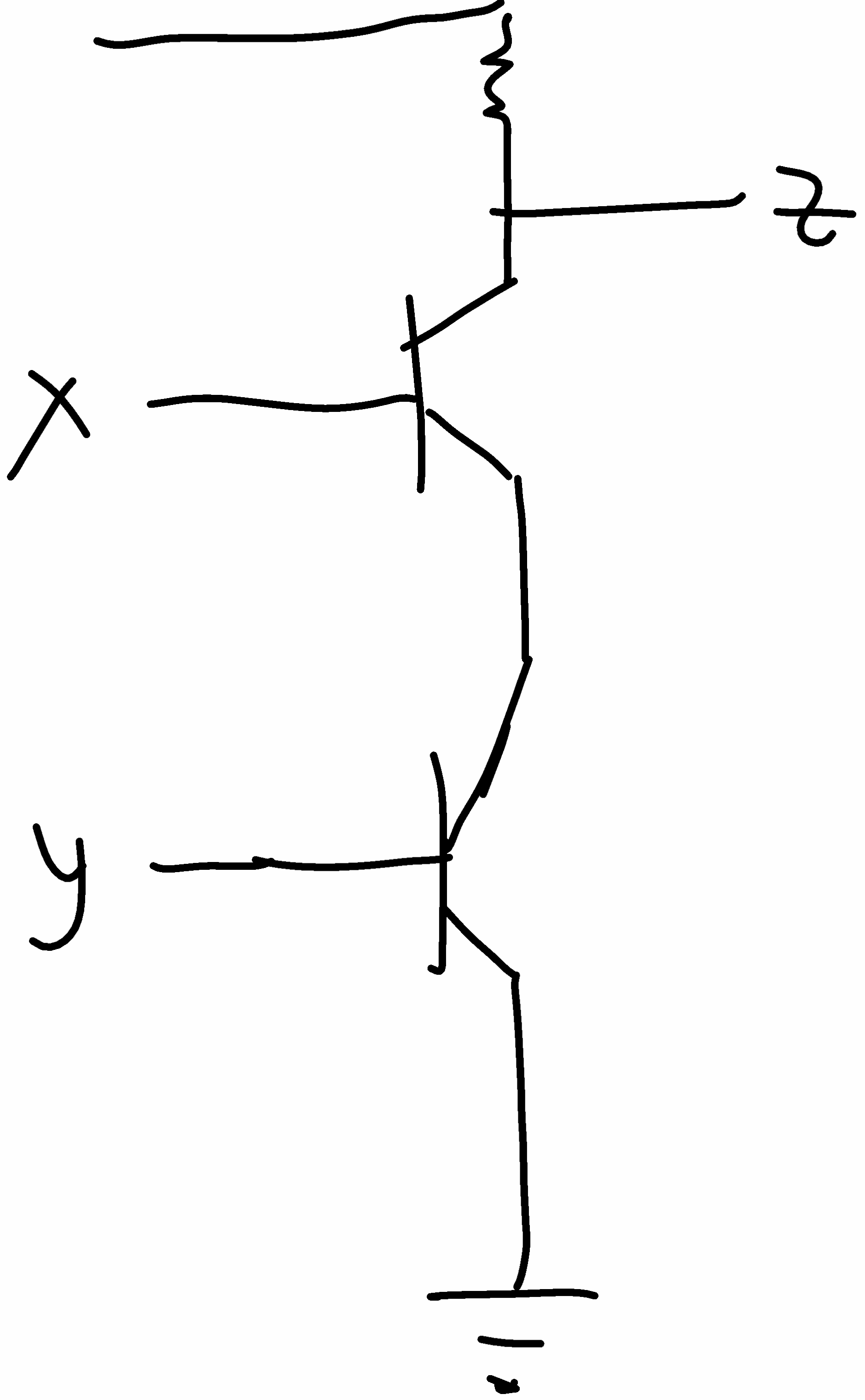




In class we saw another possibility:



11)



$z = x \text{ NAND } y = \text{NOT}(x \text{ AND } y)$