$0 + p = p$ $\|$ $((F \vee p) \leftrightarrow p)$

p	$F \vee p$	
1	1	1
0	0	1

$F \vee p$ same TT p.

p	$F \vee p$
1	1
0	0

$0 \cdot p = 0$ $\|$ $F \wedge p \leftrightarrow F$

2

$1 + p = 1$ | $(T \lor p) \leftrightarrow T$

$1 \cdot p = p$ | $T \land p \leftrightarrow p$

3

$p + pq = p$ | $p \lor (p \land q) \leftrightarrow p$

p	q	$p \land q$	$p \lor (p \land q)$	$\leftrightarrow$
0	0	0	0	1
0	1	0	0	1
1	0	0	1	1
1	1	1	1	1

3 $p \cdot (p+q) = p$

$p \wedge (p \vee q) \leftrightarrow p$

Prove that this is a tautology

p	q	$p \vee q$	$p \wedge (p \vee q)$	$\leftrightarrow$
0	0	0	0	1
0	1	1	0	1
1	0	1	1	1
1	1	1	1	1

LHS & RHS are $\equiv$ because $\leftrightarrow$ is a tautology

IMPORTANT LAWS OF PL

a) $\overline{\overline{p}} \leftrightarrow p$

b) $p \vee \neg p \leftrightarrow T$

$p \wedge \neg p \leftrightarrow F$

Ex

$(p \to q) \leftrightarrow (\neg p \vee q)$

c) $p \wedge (q \vee r) \leftrightarrow (p \wedge q) \vee (p \wedge r)$

d) $p \vee (q \wedge r) \leftrightarrow (p \vee q) \wedge (p \vee r)$

Distributive laws

e) $\overline{(p \vee q)} \leftrightarrow \overline{p} \wedge \overline{q}$

f) $\overline{(p \wedge q)} \leftrightarrow \overline{p} \vee \overline{q}$

De Morgan laws

Sol

e) $\overline{(p \vee q)} \longleftrightarrow \bar{p} \wedge \bar{q}$

p	q	$\overline{p \vee q}$	$\bar{p} \wedge \bar{q}$	$\longleftrightarrow$
0	0	1	1	1
0	1	0	0	1
1	0	0	0	1
1	1	0	0	1

In all possible cases (universes) that sentence is true. Hence, it is a Tautology

ALGEBRAIC MANIPULATIONS

Relies on an axiomatic approach.

It means:

1) We are given set of propositions (called axioms) That <u>we take as</u> <u>god-given truths</u>

2) We have rules to <u>derive</u> one new statement from previous ones. This new prop. we claim it as true if all premises are

Syllogism (Aristotle)

Several types

1) Modus Ponens

$$\frac{\begin{array}{l} p \\ p \to q \end{array}}{q}$$

2) Modus Tollens

$$\frac{\begin{array}{l} p \to q \\ \neg q \end{array}}{\neg p}$$

$$\frac{\begin{array}{c} p \\ p \to q \end{array}}{q}$$

We mean $\equiv$

$$p \wedge (p \to q) \to q \qquad (1)$$

$$p \wedge (\neg p \vee q) \to q \qquad (2 \text{ By law 11 applied on 1})$$

$$\neg(p \wedge (\neg p \vee q)) \vee q \qquad (3 \text{ By law 11 on 2})$$

$$\neg p \vee \neg(\neg p \vee q) \vee q \qquad (4 \text{ De Morgan on 3})$$

$$\neg p \vee (\neg(\neg p) \wedge \neg q) \vee q \qquad (5 \text{ De Morgan on 4})$$

Teddy Ex

$$\frac{\begin{array}{c} q \\ p \to q \end{array}}{p\,?}$$

No

$\neg p \vee (p \wedge \neg q) \vee q$ (6 By law involution on 5)

$[(\neg p \vee p) \wedge (\neg p \vee \neg q)] \vee q$ (7 By Distributive law on 6)

$[T \wedge (\neg p \vee \neg q)] \vee q$ (8 By law 2 on 7)

$\neg p \vee \neg q \vee q$ (9 By def. of $\wedge$)

$\neg p \vee T$ (10 By law 2 on 9)

q.e.d. $= T$ (11 By def of $\vee$ on 10)

Modus Ponens (Ver. 2.0)

$$\begin{array}{l} p \\ p \to q \\ \hline q \end{array} \equiv \quad p \wedge (p \to q) \longrightarrow q$$

$\neg[p \wedge (p \to q)] \vee q$ (1 By law 11 on *)

$\neg[p \wedge (\neg p \vee q)] \vee q$ (2 By law 11 on 1)

$\neg p \vee \neg(\neg p \vee q) \vee q$ (3 By De Morgan on 2)

$(\neg p \vee q) \vee \neg(\neg p \vee q)$ (4 By Commutativity)

T (5 By law Excluded 3rd)

$A \vee \neg A$

Modus Tollens

$$\begin{array}{l} p \to q \\ \neg q \\ \hline \neg p \end{array} \equiv [\neg q \wedge (p \to q)] \to \neg p$$

Cond. El

$\neg[\neg q \wedge (p \to q)] \vee \neg p$ (1 By law 11)

$\neg(\neg q) \vee \neg(p \to q) \vee \neg p$ (2 By De Morgan)

$q \vee \neg(p \to q) \vee \neg p$ (3 By Involution)

$A \vee \neg A \equiv (\neg p \vee q) \vee \neg(\neg p \vee q)$ (4 By Cond El & Commuth)

T (5 By law of excl 3rd)

ASSIGNMENT 1

1) $T \equiv$ "...tired..."

a) $H \equiv$ "...hungry..."

$T \wedge H$

b) $W \equiv$ "...whinge..."

$H \rightarrow W$

c) $N \equiv$ "...nap..."

$\neg N \leftrightarrow T$

d) $E \equiv$ "...eat..."

$$\neg(E \wedge H) \wedge (E \vee H) \equiv$$

$$\equiv (\neg E \vee \neg H) \wedge (E \vee H)$$

$$\equiv \cancel{(\neg E \wedge E)} \vee (\neg E \wedge H) \vee (\neg H \wedge E) \vee \cancel{(\neg H \wedge H)}$$

$$\equiv (E \wedge \neg H) \vee (\neg E \wedge H)$$

2) p "is equivalent to" q means $p \leftrightarrow q$ is tautology

A B $A \longleftrightarrow B$ is a tautology

$$(a+b)\cdot(c+d) = ac + ad + bc + bd$$

3) a (4) a) $(p \vee q) \vee r \longleftrightarrow p \vee (q \vee r)$

$$bd = [p \vee (q \vee r)] \wedge [(p \vee q) \vee r]$$

$$\{\neg[(p \vee q) \vee r] \vee [p \vee (q \vee r)]\} \wedge \{\neg[p \vee (q \vee r)] \vee [(p \vee q) \vee r]\} \quad (1)$$

$$\{[(\neg p \wedge \neg q) \wedge \neg r] \vee [p \vee (q \vee r)]\} \wedge \{[\neg p \wedge (\neg q \wedge \neg r)] \vee [(p \vee q) \vee r]\} \quad (2$$

By De Morgan on 1

$$[(\neg p \wedge \neg q) \wedge (\neg q \wedge \neg r) \wedge \neg p \wedge \neg r] \vee [(\neg p \wedge \neg q) \wedge \neg r \wedge \{(p \vee q) \vee r\}] \vee [\{p \vee (q \vee r)\} \wedge \neg p \wedge (\neg q \wedge \neg r)] \vee \quad (3$$

Distr., commutat. $\wedge$ $\vee$ associative

$$(p\wedge q)\wedge r \leftrightarrow p\wedge(q\wedge r)$$

$$\neg(p\wedge(q\wedge r)) \rightarrow \neg((p\wedge q)\wedge r)$$

$$\neg p\vee(\neg q\vee\neg r) \rightarrow (\neg p\vee\neg q)\vee\neg r$$

$$\neg((p\wedge q)\wedge r) \rightarrow \neg(p\wedge(q\wedge r))$$

$$(\neg p\vee\neg q)\vee\neg r \rightarrow \neg p\vee(\neg q\vee\neg r)$$

We assume $\wedge$ is associative

Hence

$(p\wedge q)\wedge r \leftrightarrow p\wedge(q\wedge r)$ is true

(1) $A \rightarrow B$
(2) $A \leftarrow B$

} By def of $\leftrightarrow$

(3) modus tollens of (1): $\neg B \rightarrow \neg A$
(4) " " " (2): $\neg A \rightarrow \neg B$
(5) De Morgan on 3
(6) " on 4
(7) Conclusion

the $\vee$ is associative