LOGIC CIRCUITS

Tue 1 Oct 2018

ADDITION

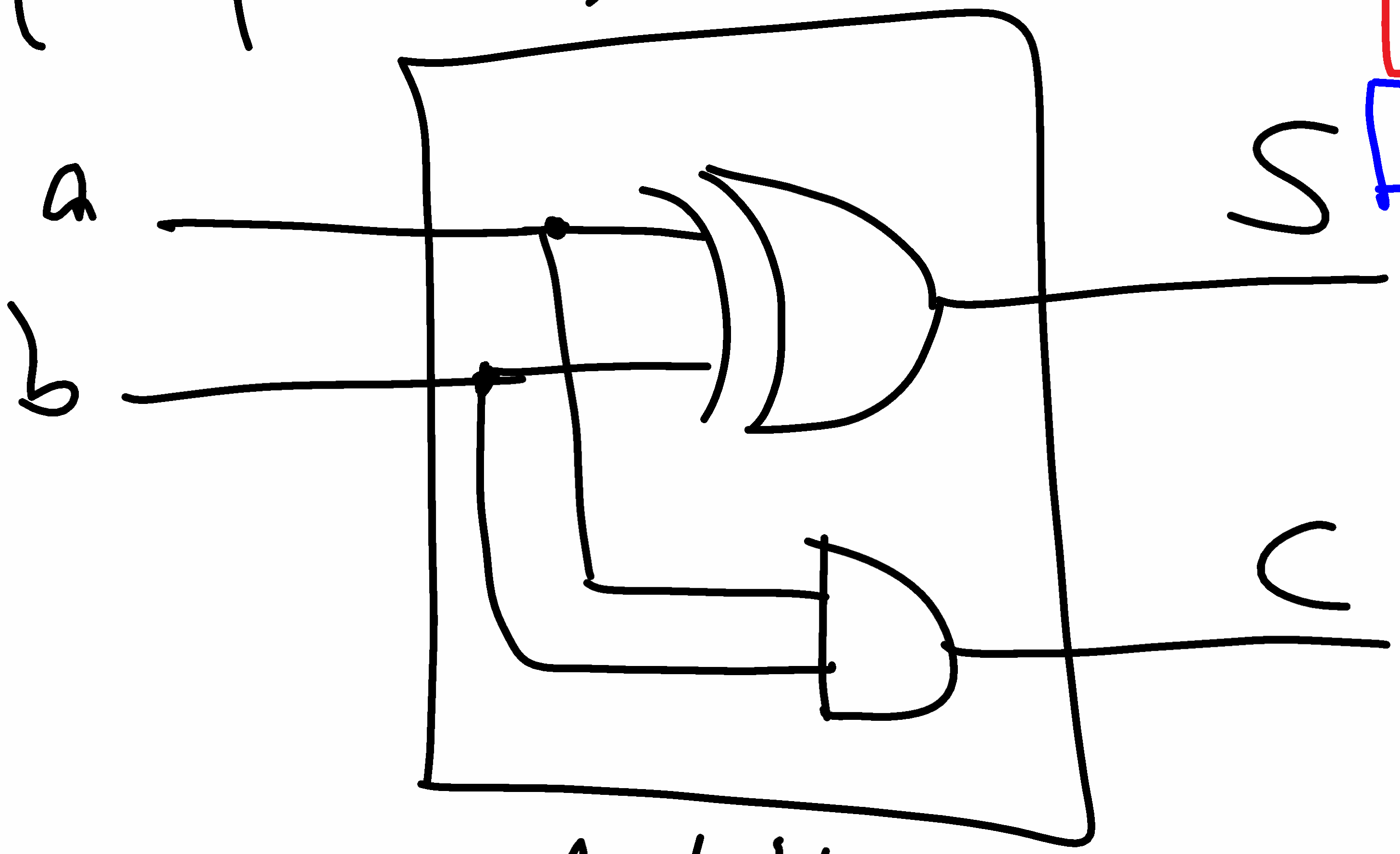
$$\begin{array}{r} 0 \\ 0\,1 \; a \\ +\;1\,0 \; b \\ \hline 1\,1 \end{array}$$

$S = a \oplus b$

$$\begin{array}{r} 1 \\ 0\,1 \; a \\ +\;0\,1 \; b \\ \hline 1\,0 \end{array}$$

$\text{Carry} = a \wedge b$

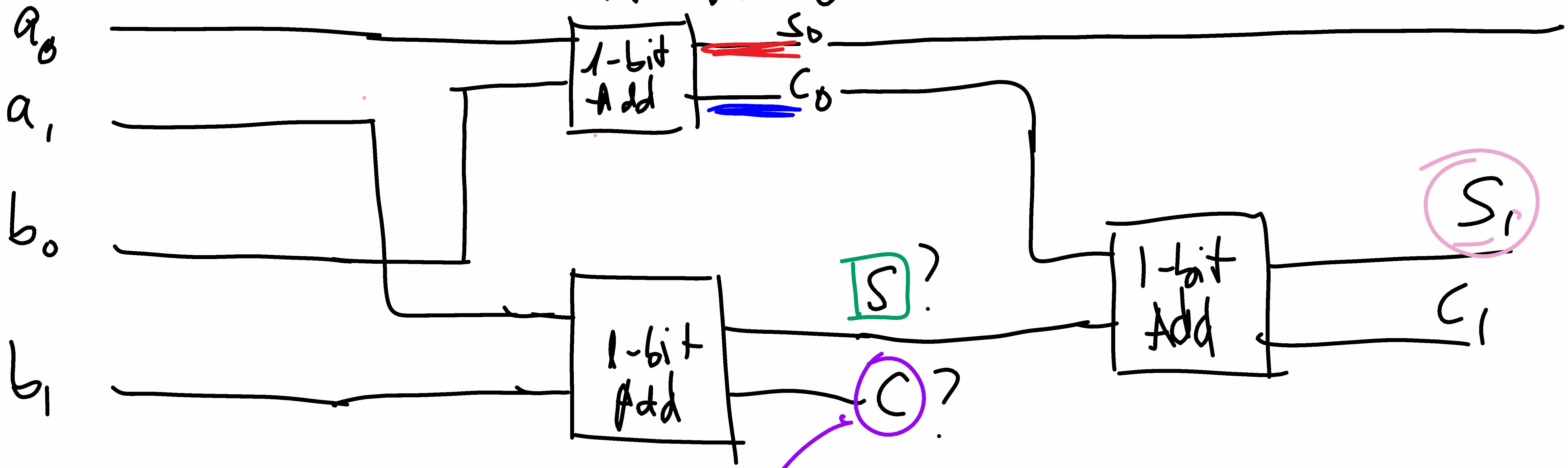
1 bit adder
(half adder)



1-bit

a	b	S	C
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

2-bit Adder



$a \in$ 0 1

$b \in$ 1 1 1 1 (c_1)

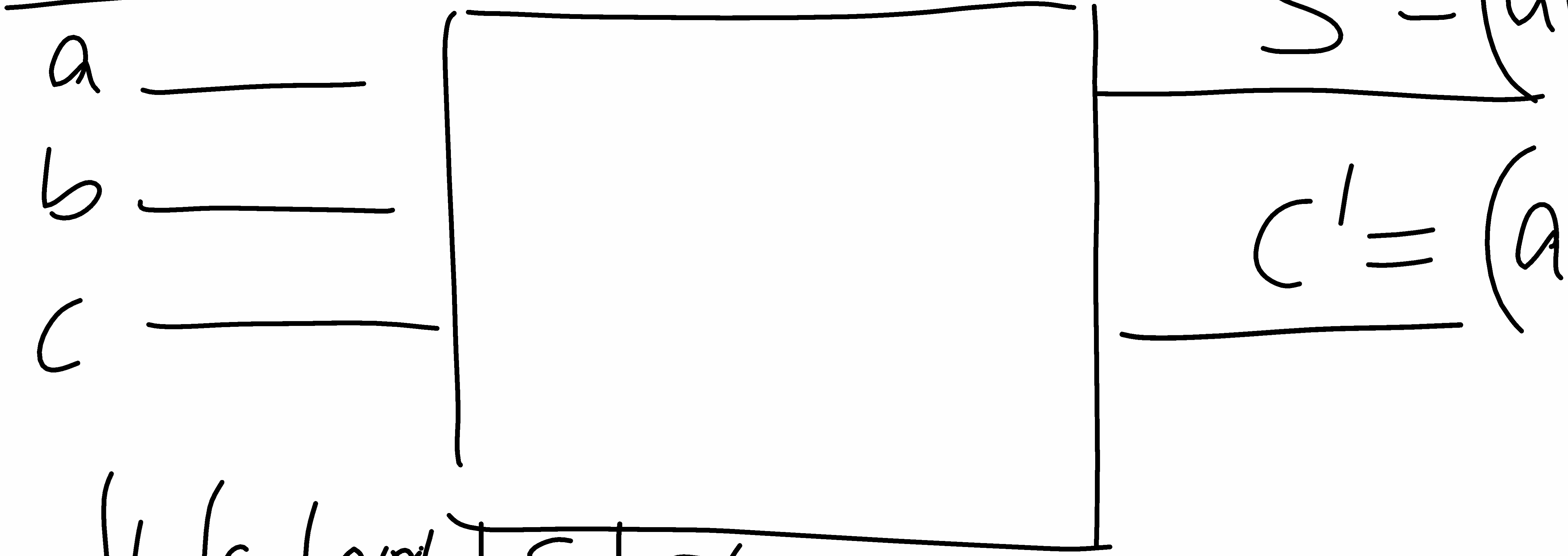
0 (s_1) 0

1 1

1 1 1 (c_0) 1

1 (s_1) 0 (s_0)

1-bit Full Adder



$$S = (a \oplus b) \oplus c$$

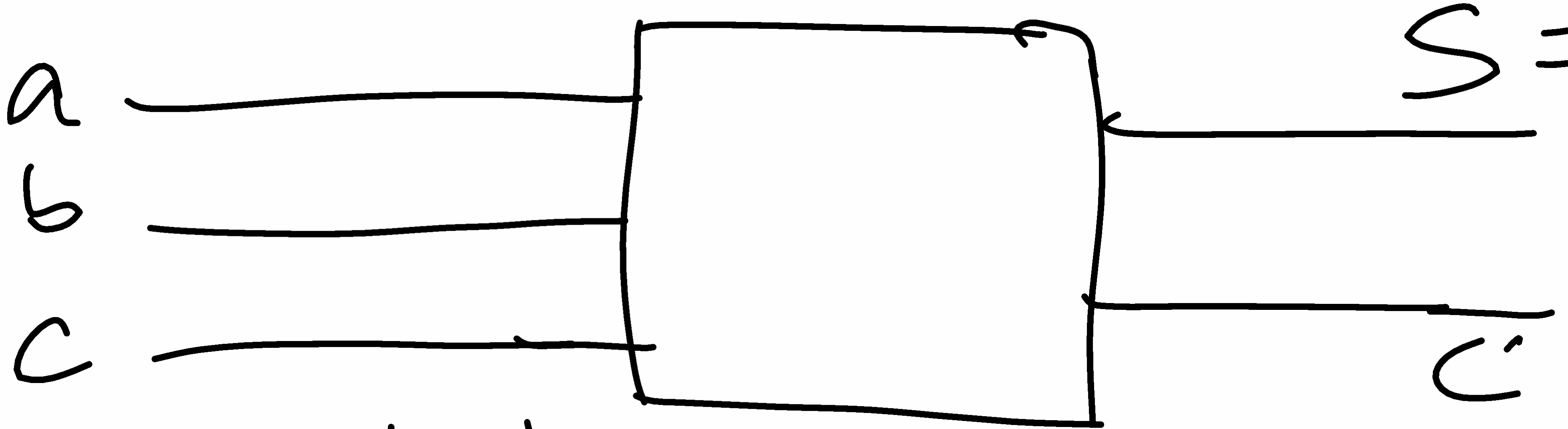
$$C' = (a \wedge b) \oplus c \;\; ??$$

a	b	c	$a \oplus b$	S	C'	
0	0	1	0	1	1	←
0	1	1	1	0	1	
1	0	1	1	0	1	
1	1	1	0	1	0	←

$$\begin{array}{r} 0' \\ 1\,1 \\ \hline 0 \end{array} \qquad \begin{array}{r} 1' \\ 1\,1 \\ \hline 1 \end{array}$$

1 bit Full Adder (renewed)

Wed 17 Oct 2018



$$S = (a \oplus b) \oplus c$$

$$c' = (a \wedge b) \oplus [(a \oplus b) \wedge c]$$

a	b	c	S	C'
0	0	0	0	0
0	1	0	1	0
1	0	0	1	0
1	1	0	0	1
0	0	1	1	0
0	1	1	0	1
1	0	1	0	1
1	1	1	1	1

$(a \oplus b) \oplus c = S$? Yes!

$a \wedge b \stackrel{?}{=} c'$ No

$[(a \oplus b) \wedge c] \oplus (a \wedge b) \stackrel{?}{=} c'$

Exercises 1

1) Prove that $p \oplus q = q \oplus p$ (Commutativity of XOR)

2) Prove that $(p \oplus q) \oplus r = p \oplus (q \oplus r)$ (Associativity of XOR)

3) Prove that $[(a \oplus b) \wedge c] \oplus (a \wedge b) \equiv (a \wedge b \wedge \neg c) \oplus ((a \vee b) \wedge c)$

4) Write in C

a) 1-bit full adder

b) 2-bit " "

c) 3-bit " "

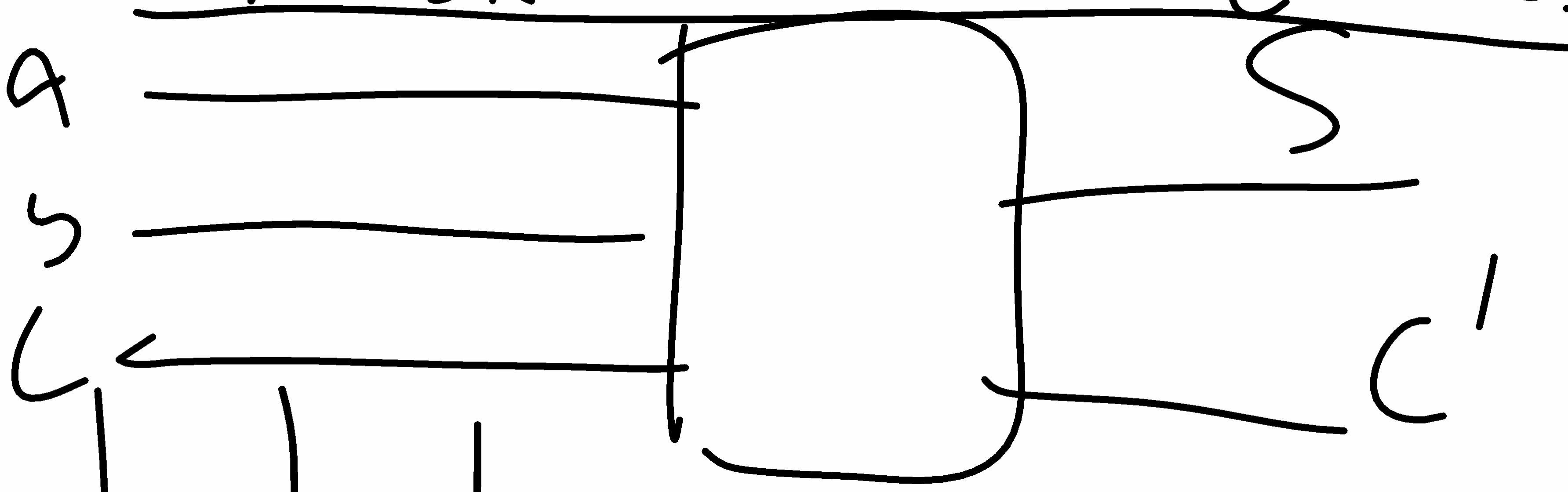
(Filename: Adders.c

(using logical operators

(see bit operations in C)

Bit operations in C: &, |, ^

1-bit Full Adder (Gabe)



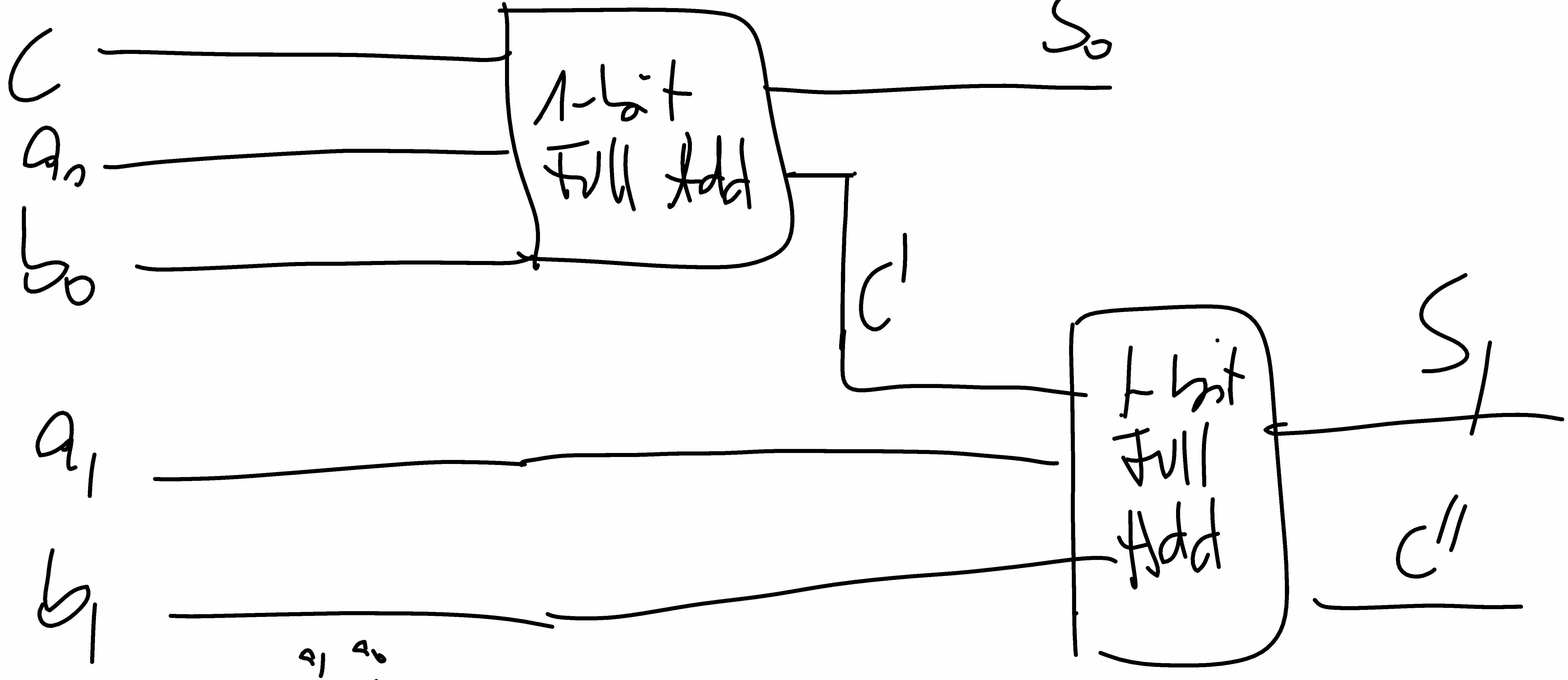
$S = a \oplus b \oplus c$

a	b	c	c'
0	0	0	0
0	1	0	0
1	0	0	0
1	1	0	1
0	0	1	0
0	1	1	1
1	0	1	1
1	1	1	1

$$c' = (a \wedge b \wedge \neg c) \oplus ((a \vee b) \wedge c)$$

$$c' = ((a \wedge b) \wedge \neg c) \oplus ((a \vee b) \wedge c)$$

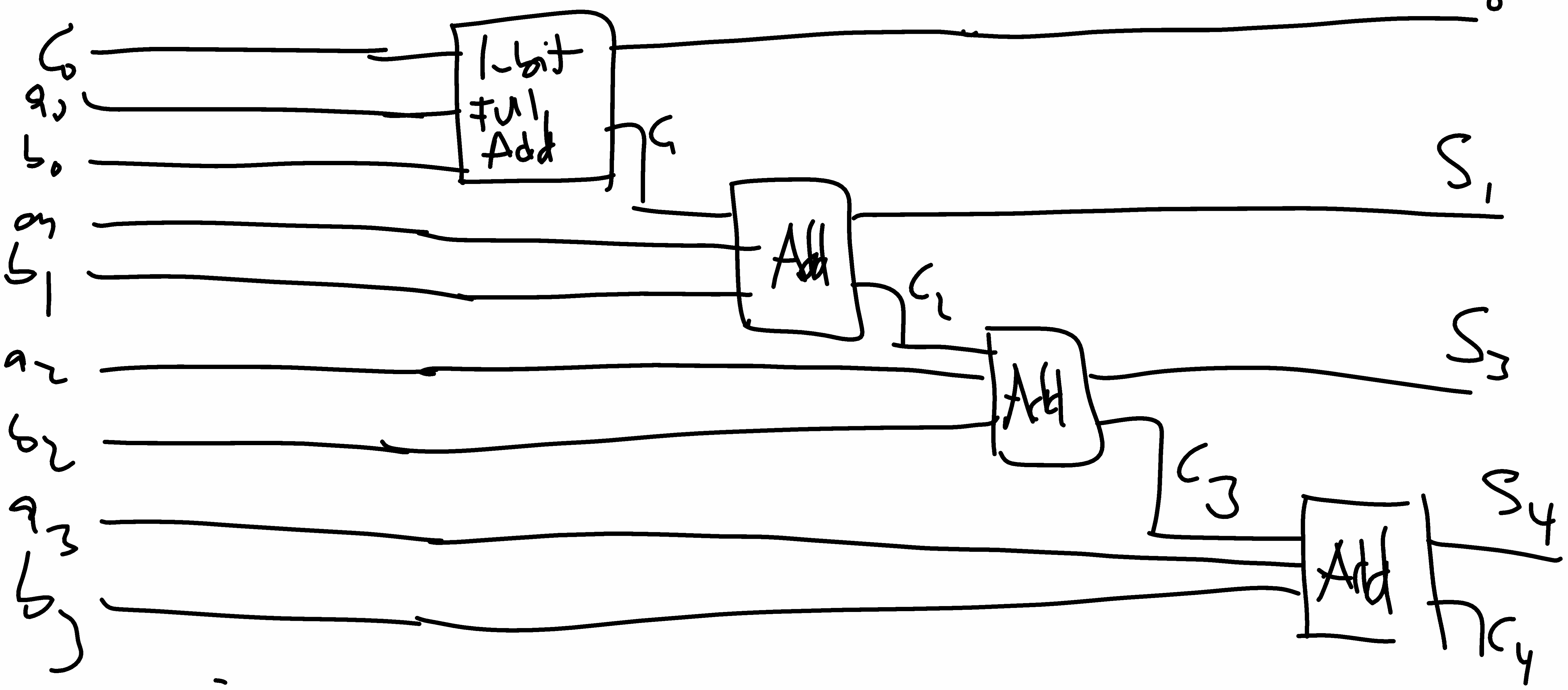
2-bit Adder



Example:

$$\begin{array}{lcc} & a_1 & a_0 \\ a = & 0 & 1 \\ b = & 1_{b_1} & 1_{b_0} \\ \hline s = & 0_{s_1} & 0_{s_0} \end{array} \quad / \; C'' = 1$$

n-bit full adder



⋮

etc.