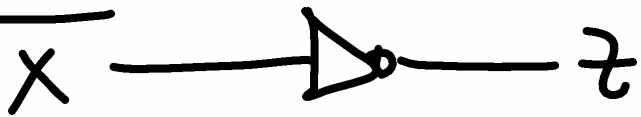


REVERSIBLE GATES

Tue 23 Oct 2018

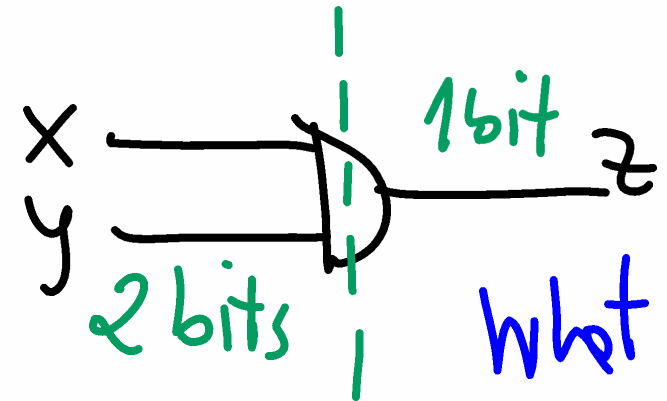
Motivation 1:

Reversible
Gate



if $z=0$, what was x ? Trivial! $x=1$

Irreversible
Gate



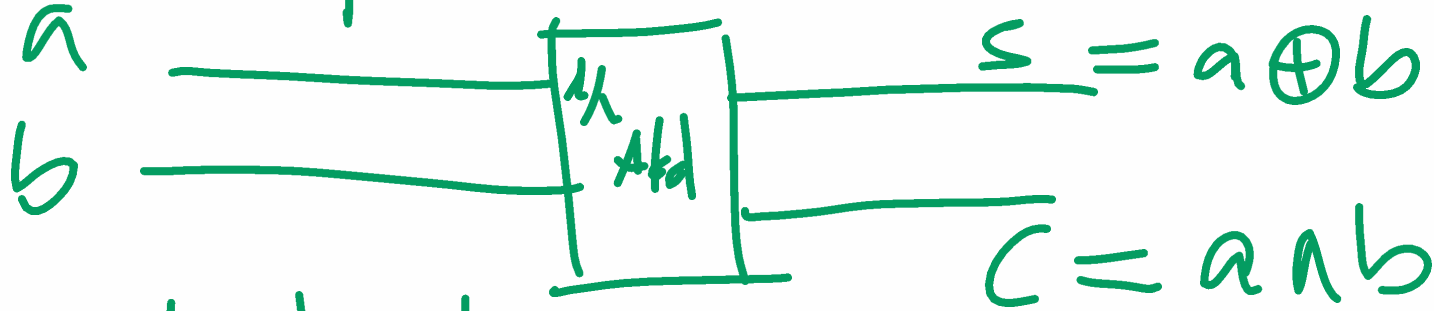
if $z=0$, what were x & y ? No way to tell

With Irreversible gates we
LOSE Information

What can we say about x & y ?
Write it down as precisely as you can.
(3min)

$$(x=0) \vee (y=0) \equiv \neg x \vee \neg y$$

Half-adder is reversible?

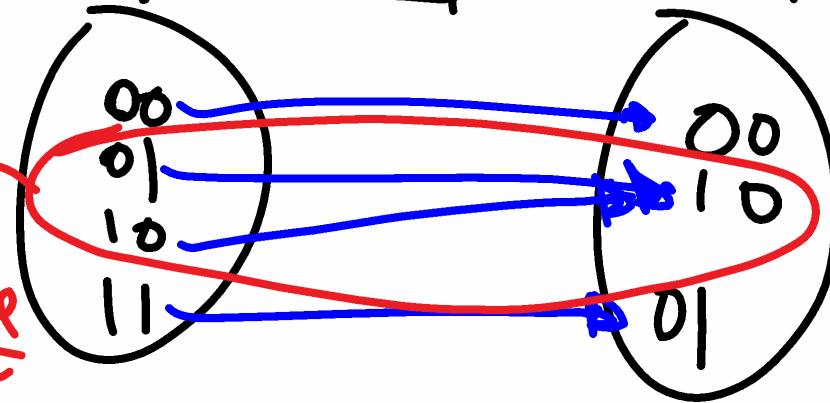


a	b	S	C
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

From Math

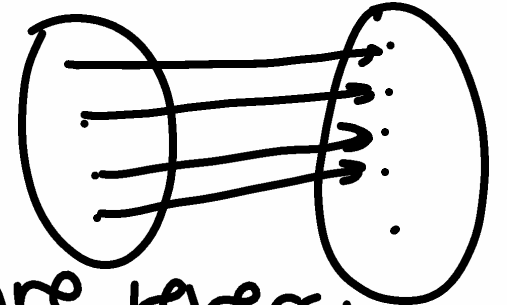
Input $\begin{bmatrix} 1/2 \\ A \end{bmatrix}$

Output



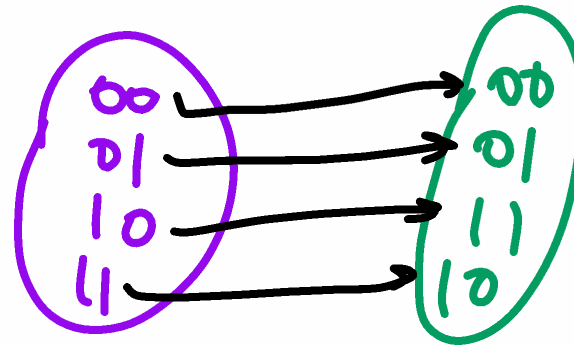
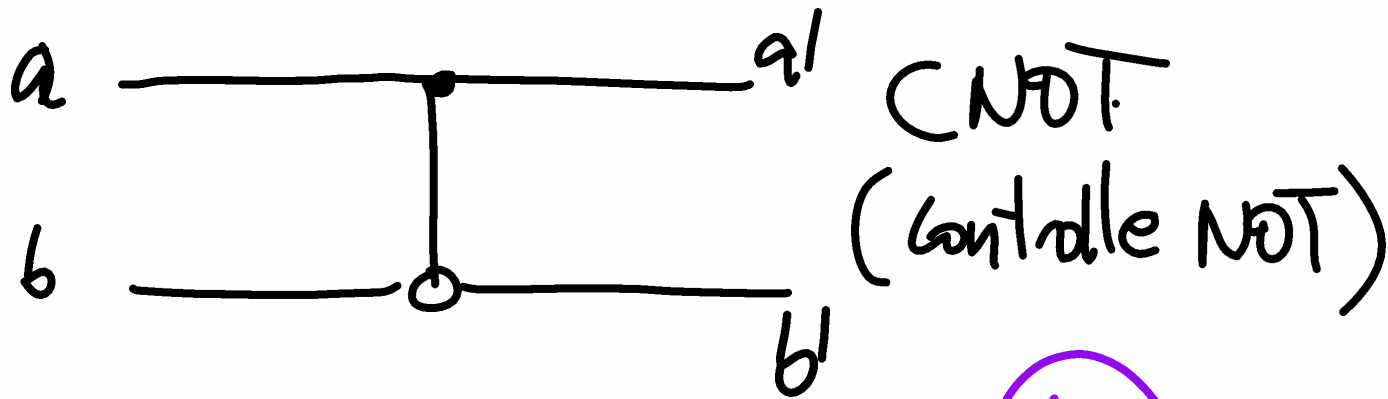
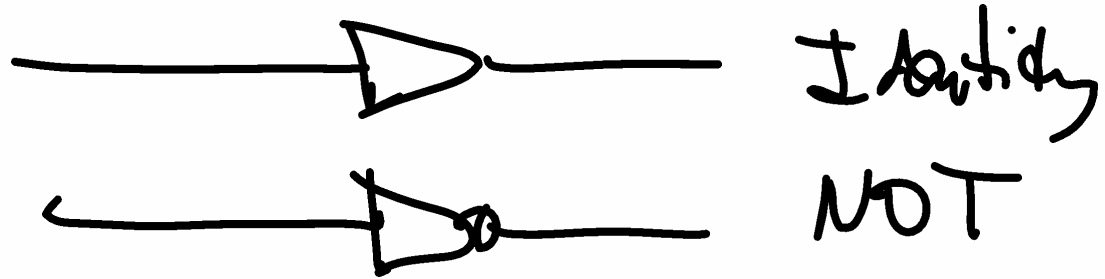
Irreversible
Function

Only functions
like (one-on-one)



are reversible

Reversible Gates

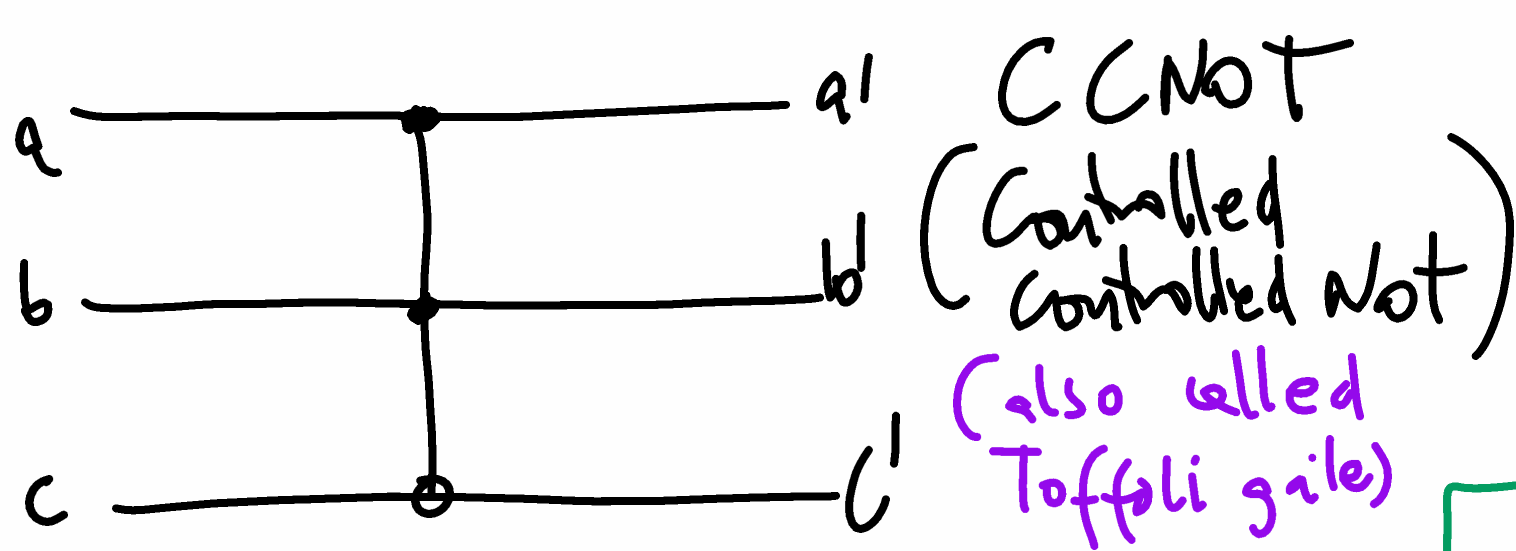


Truth table for NOT and CNOT gates:

NOT		output	
a	b	a'	b'
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

$$a' = a$$

$$b' = a \oplus b$$



Summary

$$a' = a$$

$$b' = b$$

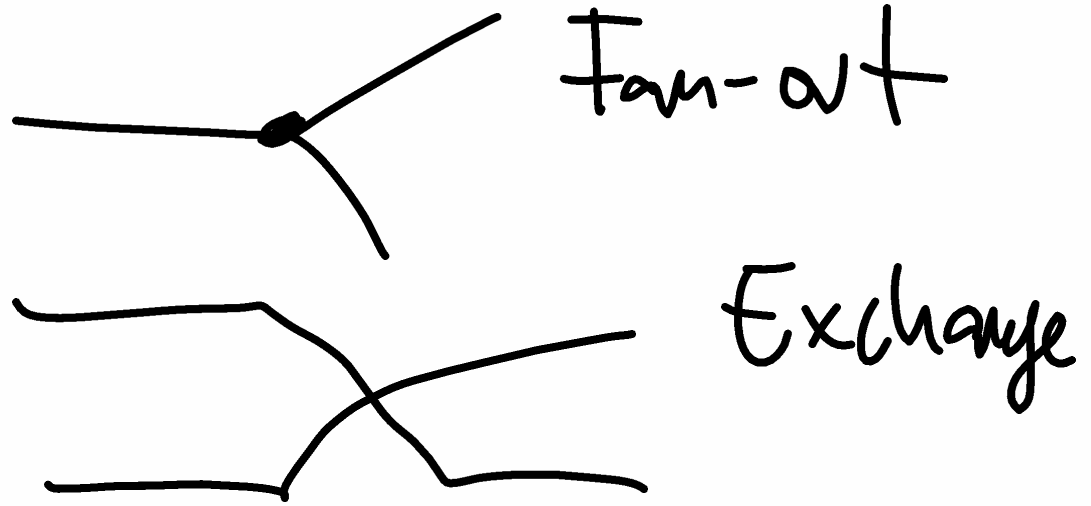
$$c' = c \text{ if } (a=b=1)$$

b. CNOT

Not

a	b	c	a'	b'	c'
0	0	0			0
0	0	1			1
0	1	0			0
0	1	1			1
1	0	0			0
1	0	1			1
1	1	0			1
1	1	1			0

of wire



both are reversible.

Homework

- 1) Build $x \text{ AND } y$ using reversible gates
- 2) Idea for $x \text{ OR } y$