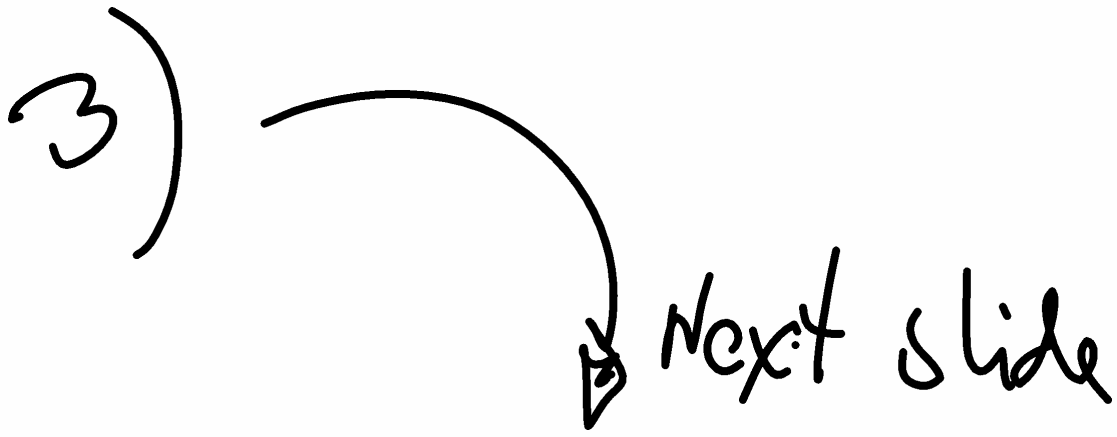
Term 1 Test 1 Solutions

Wed 24 Oct. 2018

1) c) It's Gold out there

2) $\neg F \equiv (p \vee q) \wedge (r \vee s \vee t)$

3) 

3) $f = (q \vee r \vee s) \wedge (q \vee r \vee s) \wedge (p \vee q \vee s)$

$= [(q \wedge p) \vee (r \vee s)] \wedge (p \vee q \vee s)$ [Dist. law]

$= [(q \wedge p) \wedge (p \vee q \vee s)] \vee [(r \vee s) \wedge (p \vee q \vee s)]$ [Dist. law]

$= (q \wedge p) \vee (q \wedge p \wedge s) \vee [(r \vee s) \wedge p] \vee [(r \vee s) \wedge q] \vee [(r \vee s) \wedge s]$ [Dist. law]

$\{q \wedge p\} \wedge (p \vee q) =$

$= q \wedge (p \vee (p \wedge q)) = (q \wedge p) \vee (q \wedge p \wedge q) = q \wedge p$

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$= (q \wedge p) \vee (q \wedge p \wedge s) \vee [(r \vee s) \wedge p] \vee [(r \vee s) \wedge q] \vee [(r \vee s) \wedge s]$

$= (q \wedge p) \vee (q \wedge p \wedge s) \vee (r \wedge p) \vee (s \wedge q) \vee (r \wedge q) \vee (s \wedge p) \vee (r \wedge s) \vee s$ [Dist. Law]

4) $f = [(p \vee q) \wedge r] \vee (p \wedge q \wedge r)$

$= (p \wedge r) \vee (q \wedge r) \vee (p \wedge q \wedge r)$ [Dist. law]

$= (p \wedge r) \vee (q \wedge r) = (p \vee q) \wedge (r \vee q) \wedge (p \vee r) \wedge t = (p \vee q \vee r)$ [Dist. Law]

Problem: Simplify $q \vee (p \wedge q) = q \wedge (T \vee p) = q \wedge T = q$

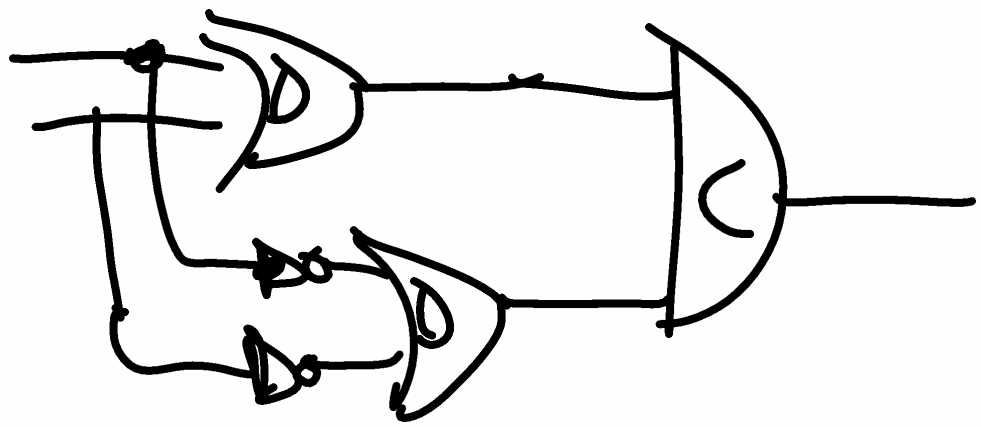
$3 + (3 \cdot 2) = 3(1+2)$

Problems

1) $\varphi \oplus \psi$ in DNF / CNF (1) (2)

(1) $(\varphi \wedge \neg\psi) \vee (\neg\varphi \wedge \psi)$

(2) $(\varphi \vee \psi) \wedge (\neg\varphi \vee \neg\psi)$



2) $p \wedge (p \vee q) \leftrightarrow p$ (1)

$p \vee (p \wedge q) \leftrightarrow p$ (2) [Dist. law]

$[\{p \wedge (p \vee q)\} \rightarrow p] \wedge [p \rightarrow \{p \wedge (p \vee q)\}]$ [Definition of Biconditional applied on 1]

$(\neg p \vee \neg(p \vee q) \vee p) \wedge [\neg p \vee (p \wedge (p \vee q))]$ [Cond Elim on 3]

$[\top \vee \neg(p \vee q)] \wedge [(\neg p \vee p) \wedge (p \vee p \vee q)]$

$\top \quad \wedge \quad \top$

$\top \wedge \top \wedge \top = \top$ q.e.d

3) "If it is January, then it is cold" $\equiv p \rightarrow q$

Hence $p \equiv$ "It's January" & $q \equiv$ "It's cold"

a1) $\neg q \rightarrow \neg p \equiv$ "If It's not cold, then it's not January"

a2) $p \wedge \neg q \equiv$ "It's January and not cold"

a3) $\neg p \rightarrow \neg q \equiv$ "If it's not January, then it's not cold"

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b) "If $y+5 \neq 7$ then $y < 0$" $\equiv p \rightarrow q$

Hence $p \equiv$ "$y+5 \neq 7$" and $q \equiv$ "$y < 0$"

b1) $\neg q \rightarrow \neg p \equiv$ "If $y \geq 0$ then $y+5 = 7$"

b2) $p \wedge \neg q \equiv$ "$y+5 \neq 7$ and $y \geq 0$"

b3) $\neg p \rightarrow \neg q \equiv$ "If $y+5 = 7$, then $y \geq 0$"