BASIC CONCEPTS OF QUANTUM COMPUTATION

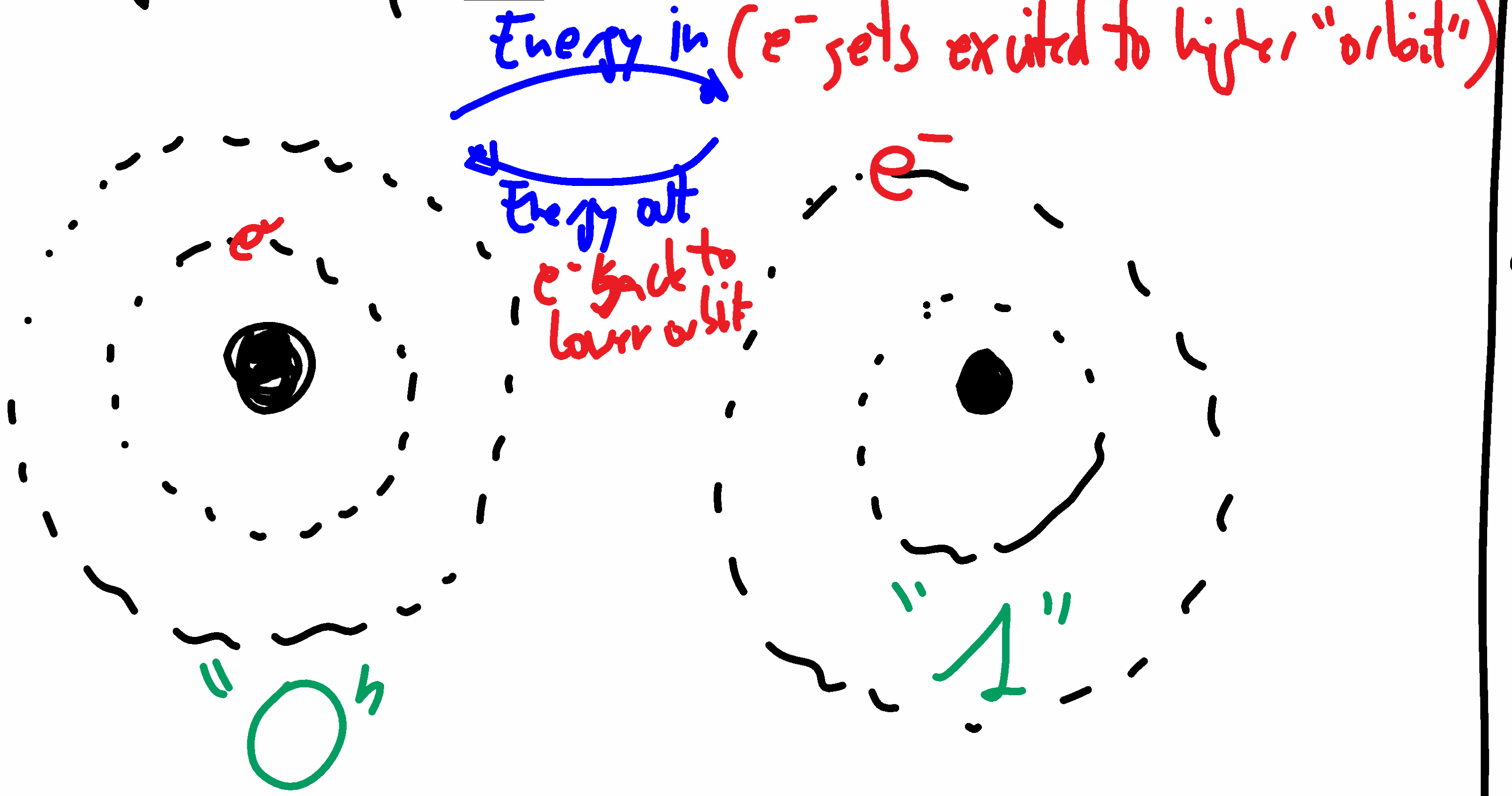
Tue 30 Oct 2018

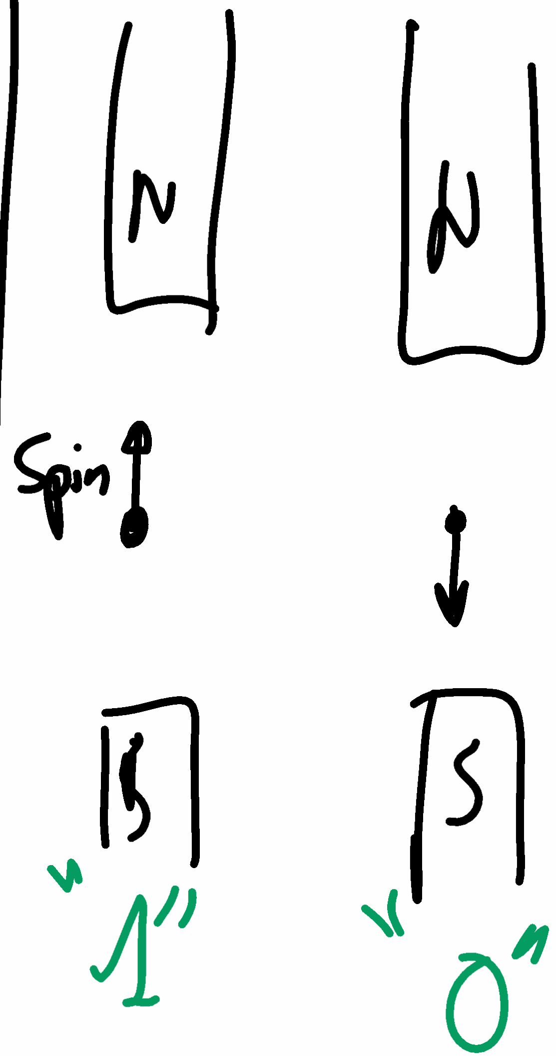
Fundamental ingredients of Classical Computers/Computation:

- Distinction of High/Low voltage (The 0's & 1's "States")
- Switch On/off (transistor)

THE QUANTUM WORLD

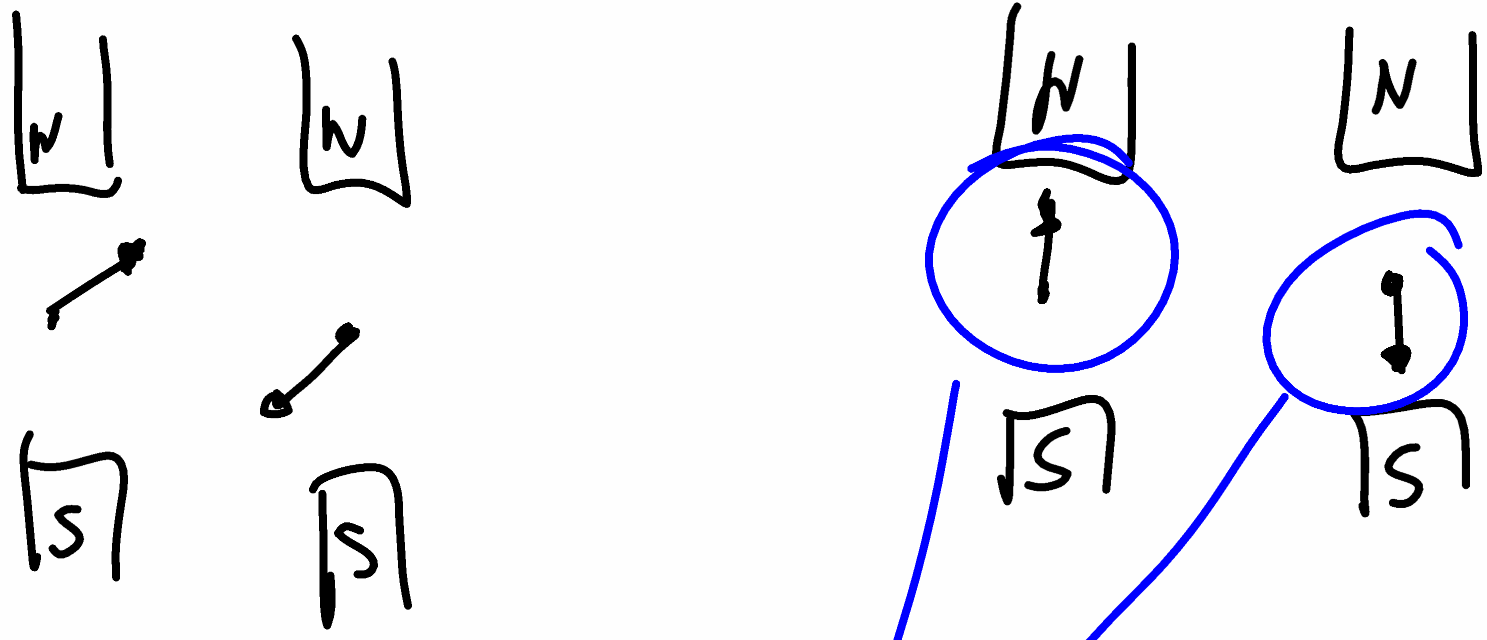
Examples of "2-values"





BASIC CONCEPTS (FACTS) OF THE QUANTUM WORLD

1) A QUANTUM SYSTEM CAN BE IN MANY STATES

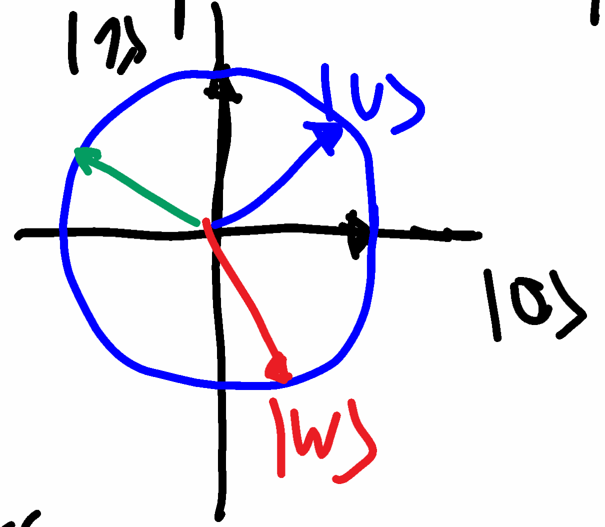


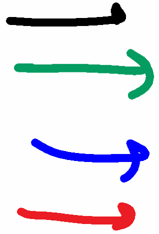
2) There is ALWAYS some FUNDAMENTAL STATES. They are states that never change if we do not interact with system

3) ALL STATES ARE VECTORS OF LENGTH = 1

THE FUNDAMENTAL STATES ~ PERPENDICULAR DIRECTIONS

<u>Example</u>: Description of spin system





≠ states

<u>Notation</u>: 0, 1, 2, ... values

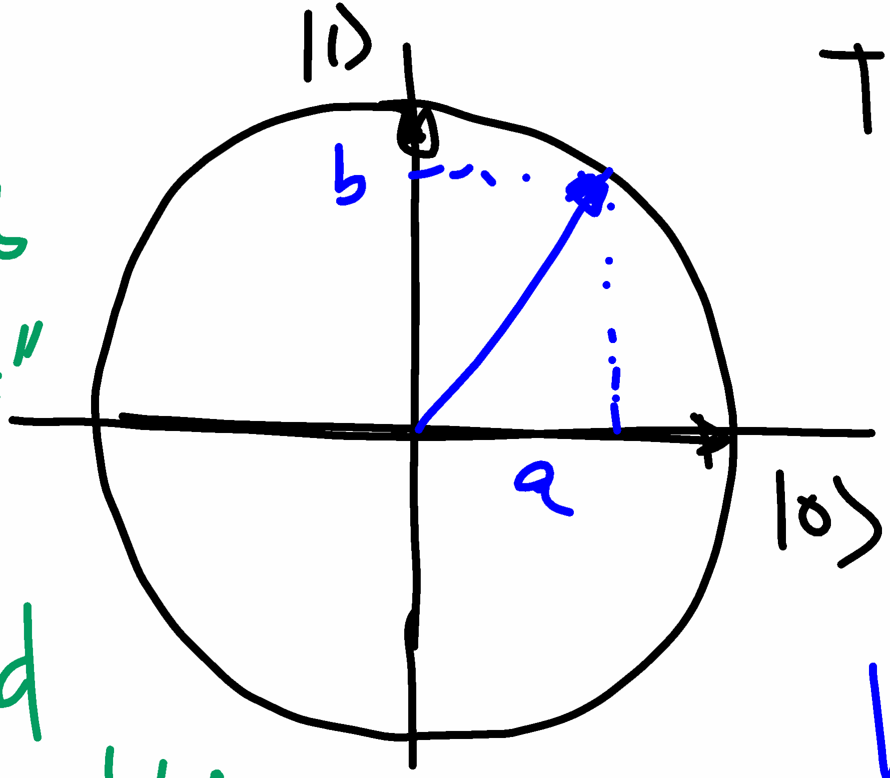
$(\vec{v})$ $|0\rangle, |1\rangle$ ... vectors

4) QUANTUM SUPERPOSITION (Schrödinger's cat)

State $|V\rangle = a|0\rangle + b|1\rangle$

Actually, a & b are "Complex #" not real #'s.

a, b are called "probability amplitudes"



This is the same as in math when we describe a point (or a vector) by it's coordinates

(a, b)

V is in a "superposition" of states $|0\rangle$ & $|1\rangle$

5) When we measure a Quantum System $|v\rangle$

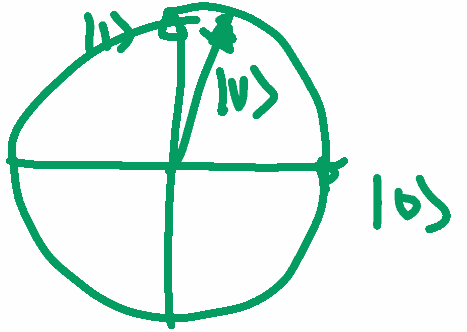
We only see 2 possible values, $|0\rangle$ or $|1\rangle$!!

$$|v\rangle = a|0\rangle + b|1\rangle$$

a^2 is the probability of measuring $|0\rangle$

b^2 " " " " " " " $|1\rangle$

Example: $|v\rangle = 0.2\,|0\rangle + x\,|1\rangle$



1) Find x^2

2) What is the probability of finding the system in $|0\rangle$

3) Idem $|1\rangle$

Sol: We know that length of $|v\rangle$ must be 1

$(0.2)^2 + x^2 = 1$ $\qquad x^2 = 1 - 0.04 = 0.96$

is $x \overset{?}{=} +\sqrt{0.96}$

or $x = -\sqrt{0.96}$

Need Grad Course

6) We can "pile up" Quantum systems in 2 ways

a) Independently

b) ENTANGLED

Examples below

7 **Qubit** : Atom (or Q syst) that has 2 fundamental states accessible
(or more)

QUANTUM CIRCUITS

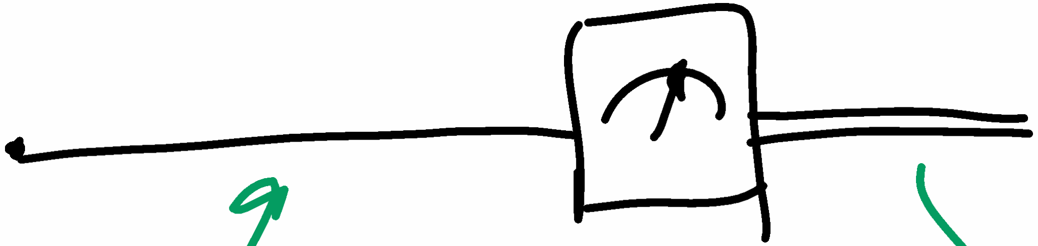
Each line is a Qubit

$\longrightarrow$ horizontal axis $\sim$ "time"

Example

1-single qubit

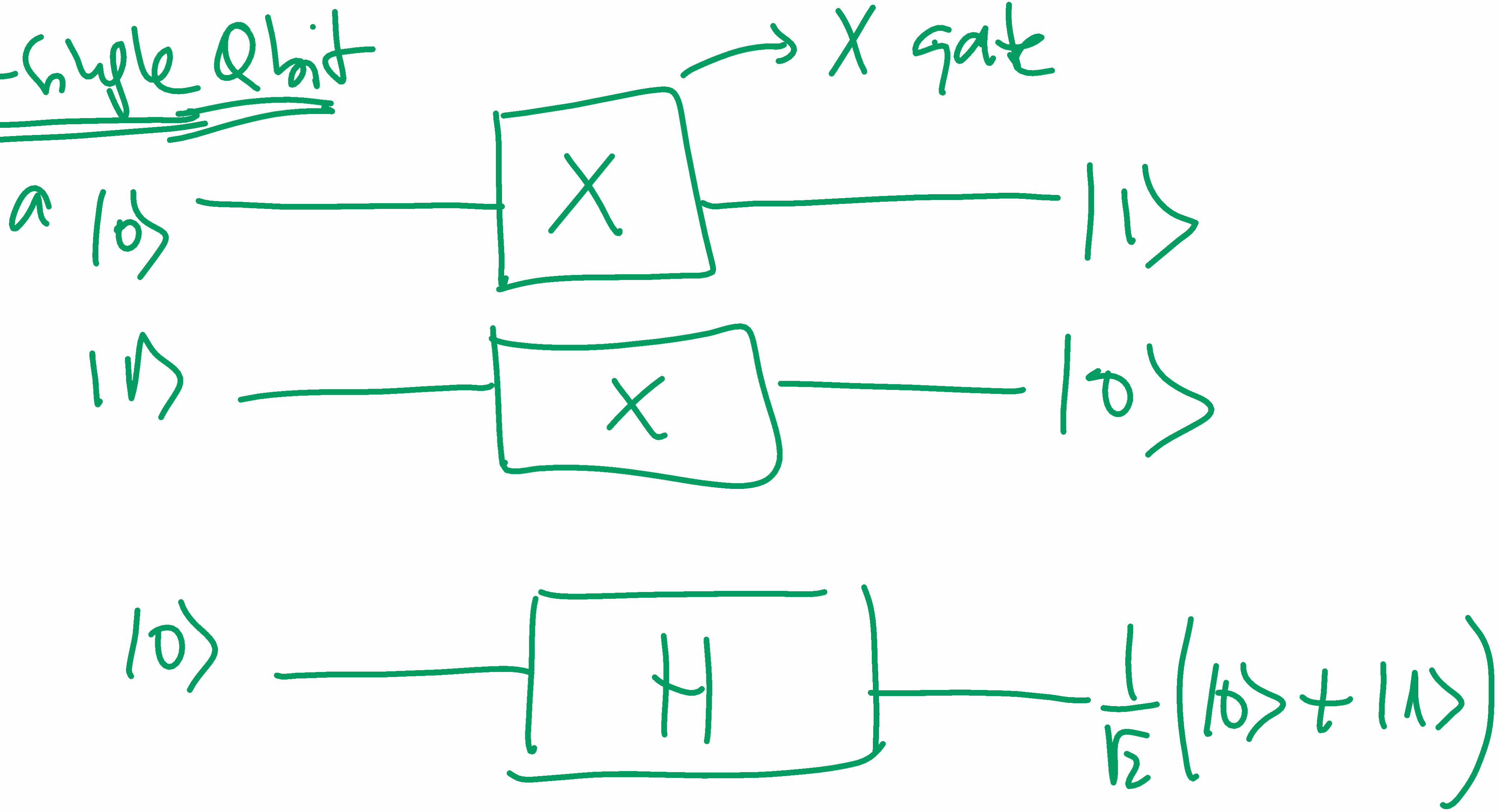
a



Q State ($|\psi\rangle$)

Classical state (i.e., $|0\rangle$ or $|1\rangle$)

1-Single Qbit



Lesson II Quantum Computing

Wed 31 Oct 2018

(· Review yesterday)

· Rule 8 Quantum World

When measuring, any state $|w\rangle = -|w\rangle$

We cannot detect an overall sign change

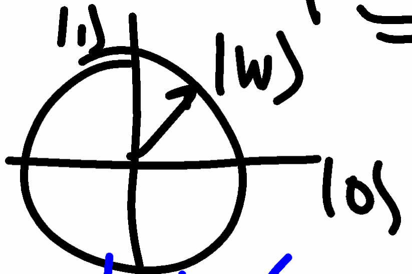
QUANTUM GATES & CIRCUITS

Q •———————— ≡ Identity Gate

atom w/ Qbit

For 1-classical bit ⇒ only 2 gates: · Identity
· NOT

For 1-Qbit ⇒ Infinite many gates

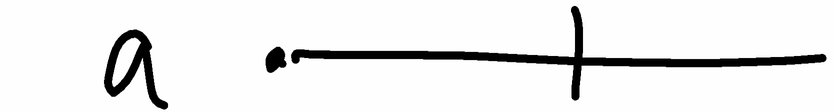


→ all rotations of $|\psi\rangle$

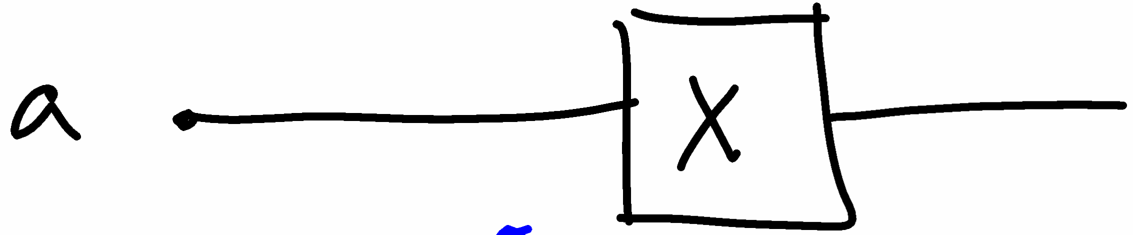
But all 1-Qbit gates can be built from only 4 gates

"Actual Q Comp use a limited set of Gates

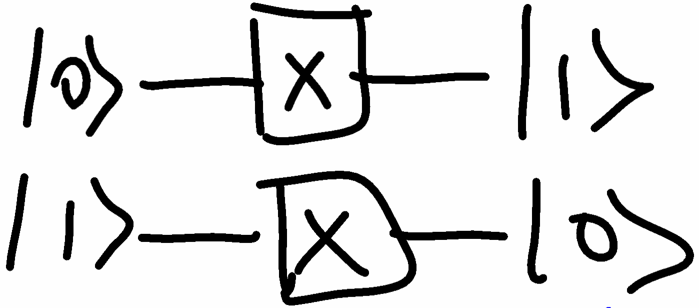
1-Qbit Gates



$\equiv$ Identity



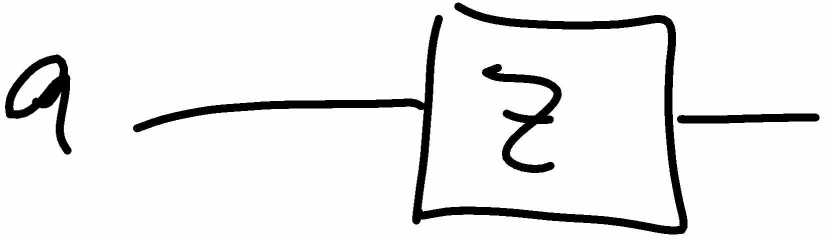
Q-NOT



Using math we write this as

$X|0\rangle = |1\rangle$

$X|1\rangle = |0\rangle$



?

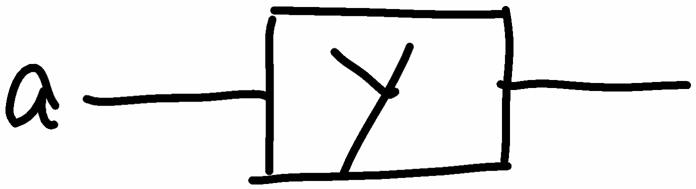
$Z|0\rangle = |0\rangle$

$Z|1\rangle = -|1\rangle$

Notation similar to function notation

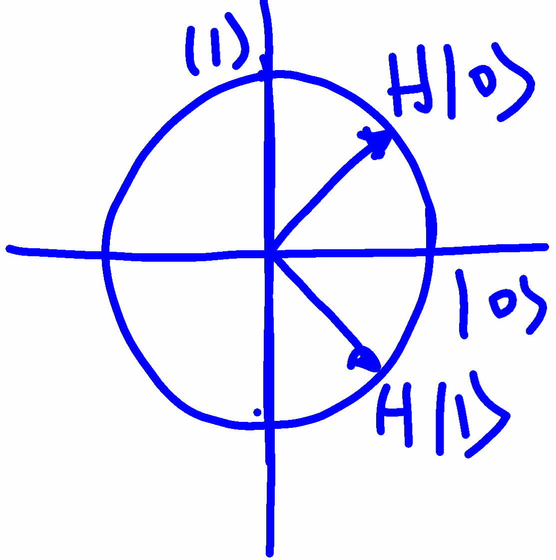
$f(0)=1 \iff X|0\rangle = |1\rangle$

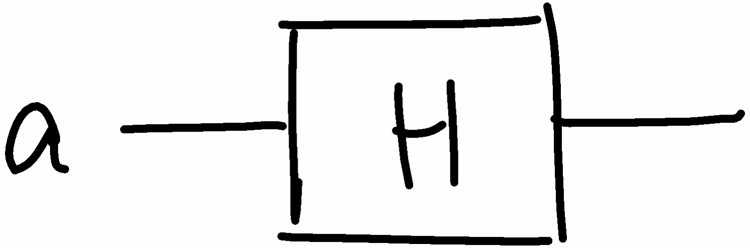
$$\left(i \equiv \sqrt{-1} \; ; \; i^2 = -1 \right)$$



$$|0\rangle = i\,|1\rangle$$

$$Y|1\rangle = -i\,|0\rangle$$





Hadamard

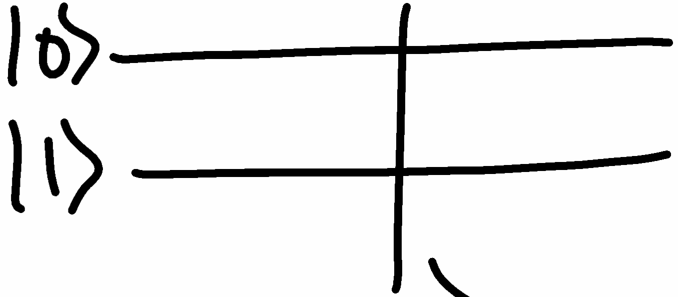
$$H|0\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

$$H|1\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)$$

ENTANGLEMENT

Comes from "piling up" Qbits $\Rightarrow$ Need 2 or more Qbits

- Independent "piling"



The state of the whole system

4 possibilities

$|0\rangle \; |0\rangle \; |1\rangle \; |1\rangle$

$|0\rangle \; |1\rangle \; |0\rangle \; |1\rangle$

2 ways to write independent Qbits

$|1\rangle|0\rangle \equiv |10\rangle$

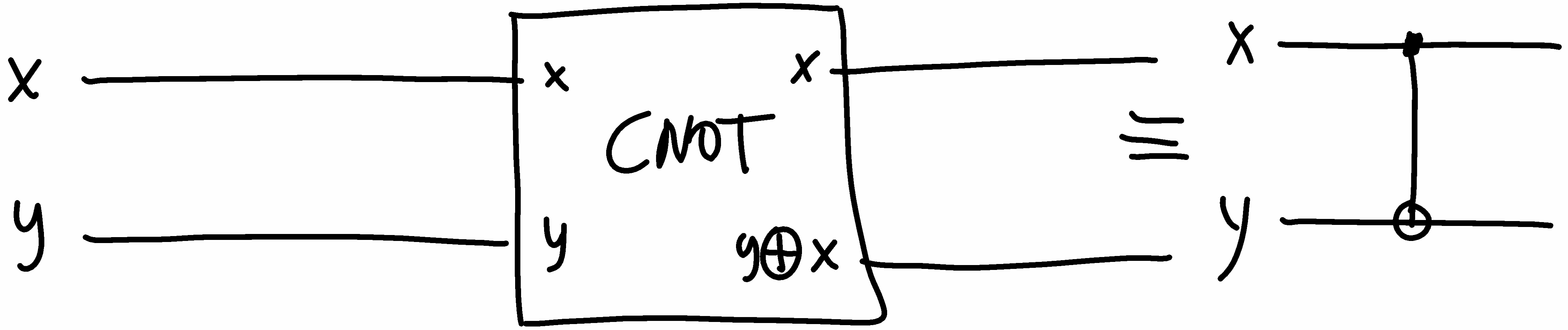
$|00\rangle$

$|01\rangle$

$|10\rangle$

$|11\rangle$

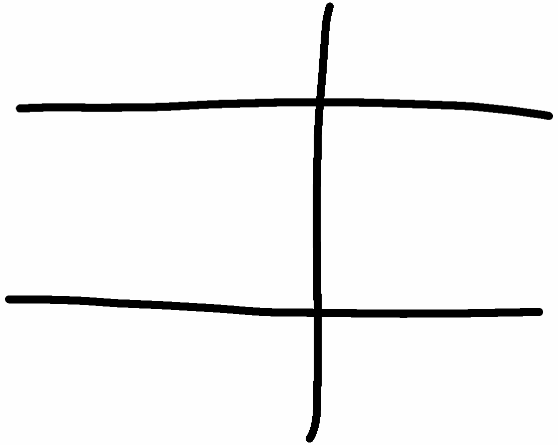
• ENTANGLED PAIR (not independent "piling")



$$|00\rangle \longrightarrow |00\rangle$$
$$|01\rangle \longrightarrow |11\rangle$$
$$|10\rangle \longrightarrow |10\rangle$$
$$|11\rangle \longrightarrow |01\rangle$$

$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

$|1\rangle$



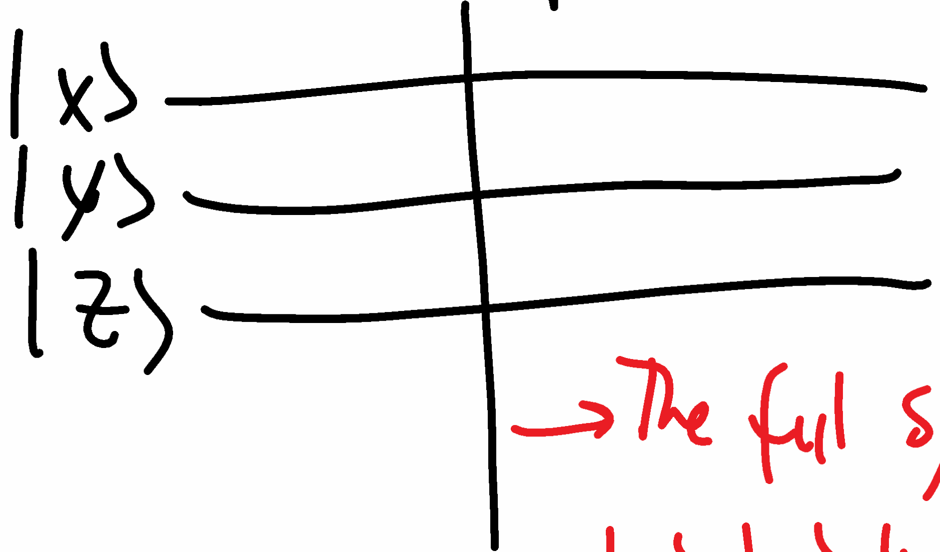
$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|1\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle|1\rangle + |1\rangle|1\rangle\right)$$

$$= \frac{1}{\sqrt{2}}\left(|01\rangle + |11\rangle\right)$$

Entanglement (Cont.)

Wed 7 Nov 2018

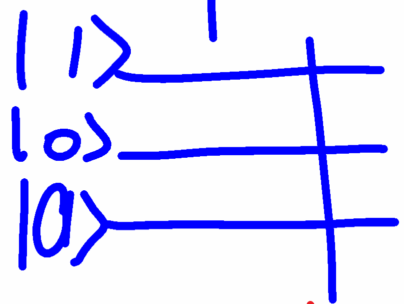
When we pile/stack qubits we represent this system by using multiple lines



→ The full system of 3 qubits

$$|z\rangle|y\rangle|x\rangle = |zyx\rangle$$

Example



1st qbit is in 1

3rd qbit in 1

$$|001\rangle = |0\rangle|0\rangle|1\rangle$$

2nd qbit is in $|0\rangle$

Independent Qbits!!

Exercise: Write the diagram corresponding to the

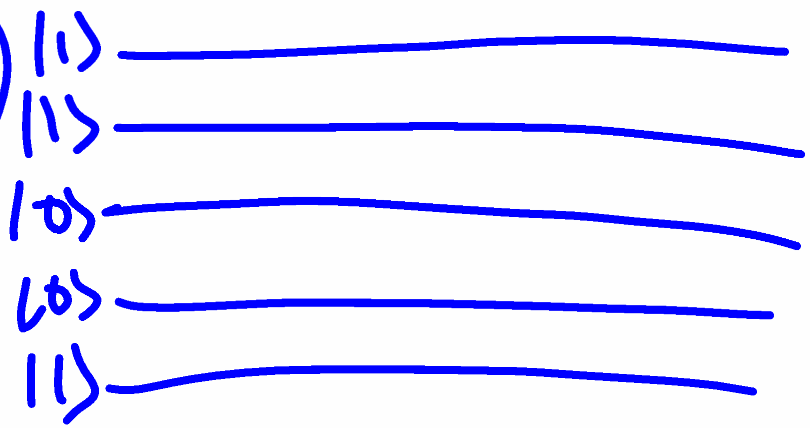
a) state $|10011\rangle$

b) How many qubits do we have?

c) Are they all independent? Why?

Sol:

a)



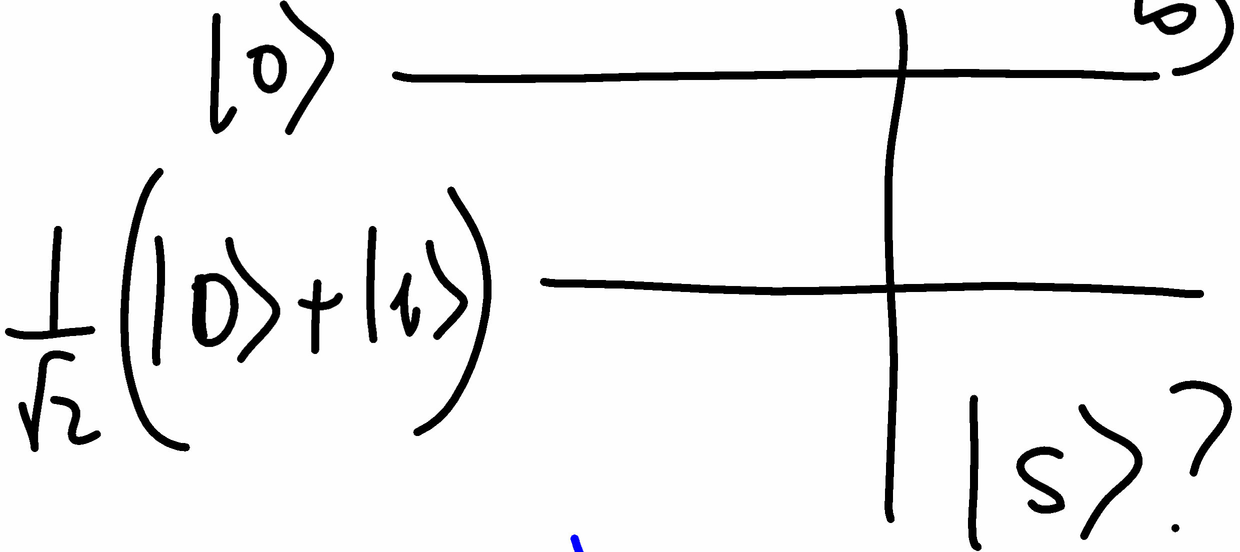
b) 5

c) Yes, because we can write the state as a product of individual qubits

$$|10011\rangle = |1\rangle|0\rangle|0\rangle|1\rangle|1\rangle$$

Exercise 2: a) Write the state of this 2-qubit system.

b) Simplify & write it in terms of the fundamental states of this system. Hint: $|00\rangle, |01\rangle, |10\rangle, |11\rangle$



c) Are these qubits indep.?

Sol: a)

$$|s\rangle = \boxed{\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)}\ \boxed{|0\rangle}$$

B: bottom qubit

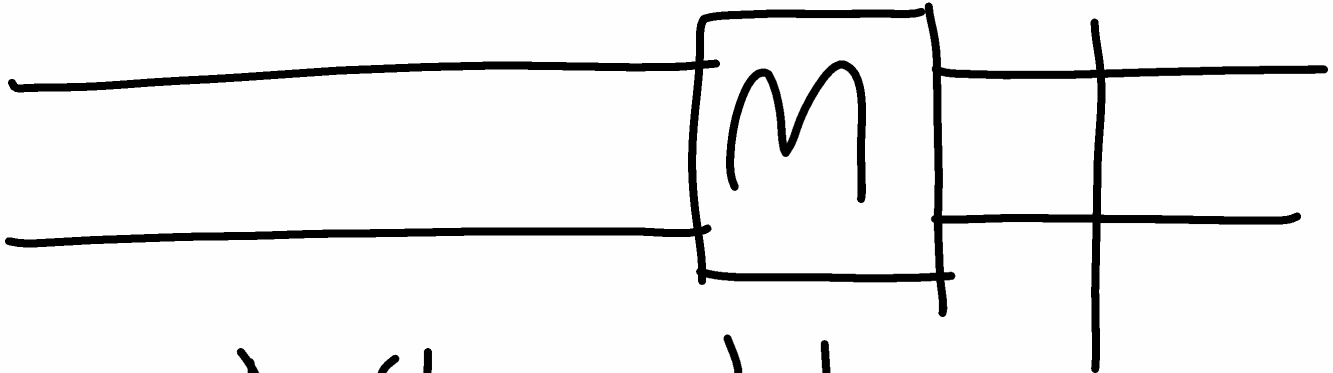
T: top qubit

b)

$$|s\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle|0\rangle + |1\rangle|0\rangle\right) = \frac{1}{\sqrt{2}}\left(|00\rangle + |10\rangle\right)$$

Yes, of course! 'cause $|s\rangle = |B\rangle|T\rangle$

Entangled Pair of Qubits



Are this qubits independent? $|S\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$

Answer: In order to be independant it should be possible to write $|S\rangle$ as a product of 1 single qubit states $|B\rangle|T\rangle$

(cont)

ANSWER: THE ARE ENTANGLED
that is not independent

1- Single qubit can be in many different states, e.g.

$|0\rangle$, $|1\rangle$, $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$, $\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$, ...

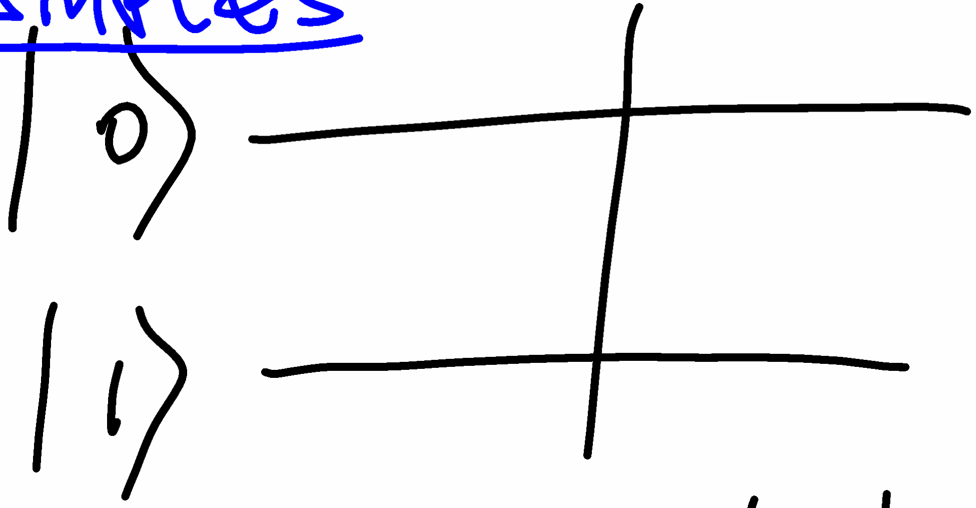
Comment: In math this problem is simmilar to

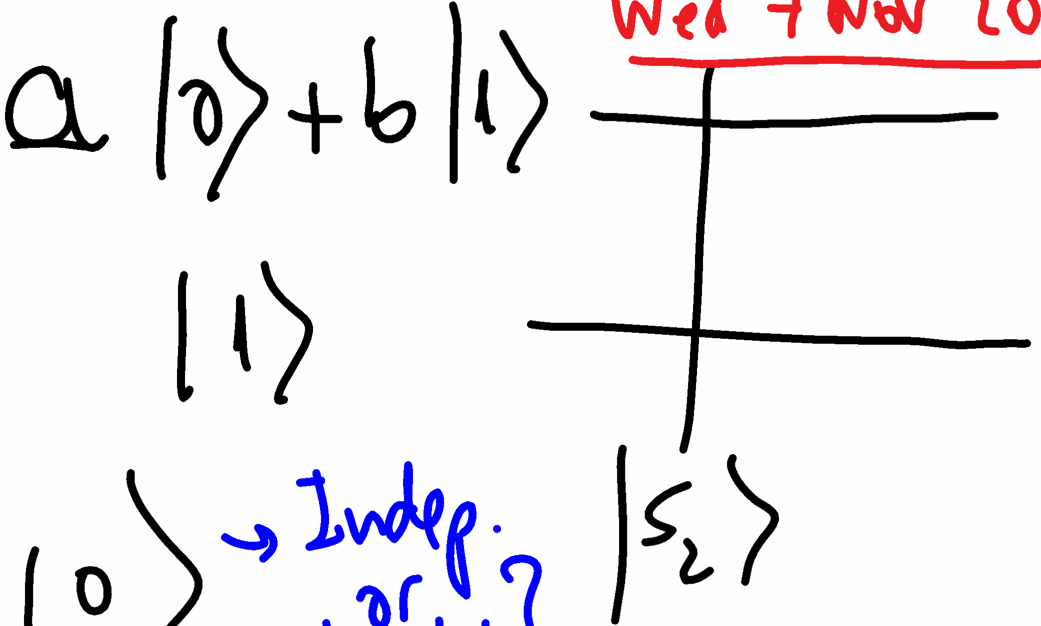
$$7 + 5 = x\, y$$

Find x & y

Wed 7 Nov 2018

EXAMPLES

(A) 

(B) 

(A) $|s_1\rangle = |1\rangle|0\rangle = |1\,0\rangle$ → Indep. or Entangled ?

Indep

(B) $|s_2\rangle = |1\rangle\big(a|0\rangle + b|1\rangle\big) = a|1\rangle|0\rangle + b|1\rangle|1\rangle =$

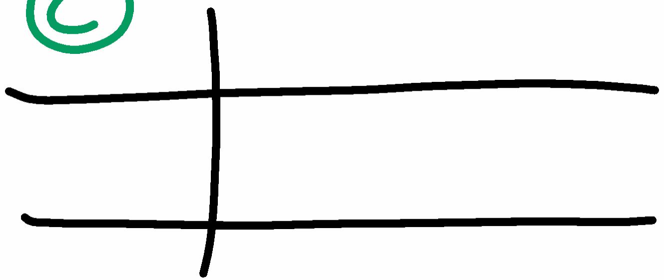
$= a|10\rangle + b|11\rangle$

Independent

(C)

$\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

$|0\rangle$



$$|S_1\rangle = |0\rangle \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right) = \frac{1}{\sqrt{2}}\left(|00\rangle - |01\rangle\right)$$

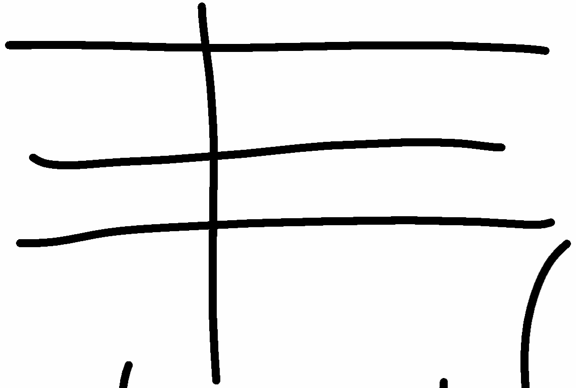
Indepent

(D)

x

y

z

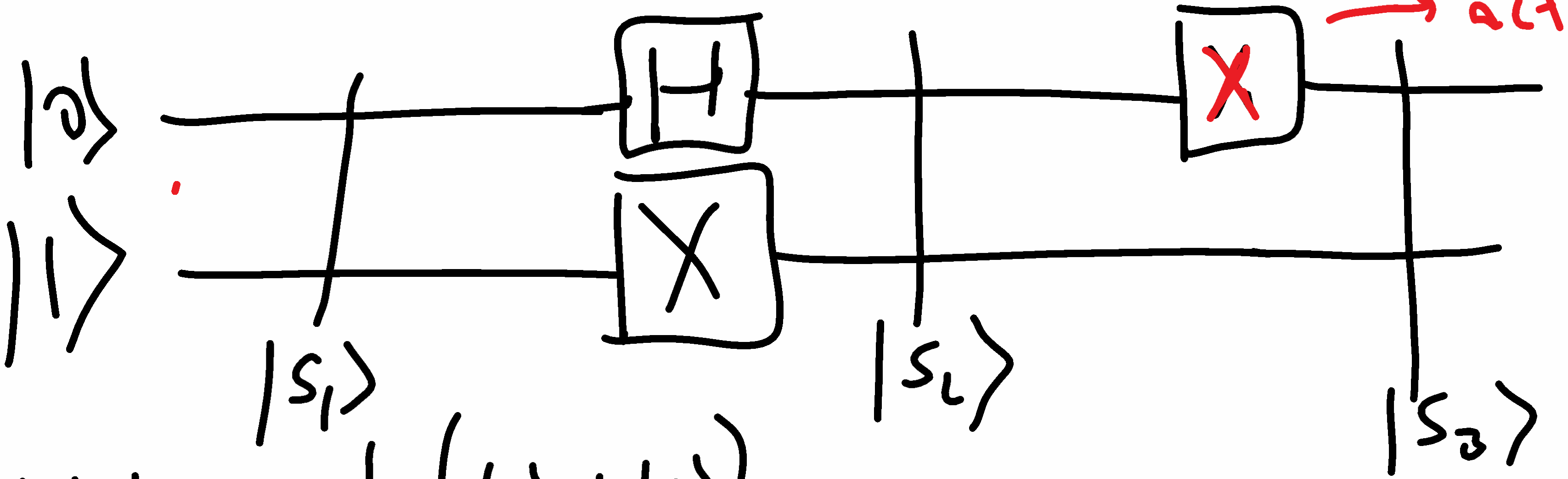


Independent

$|1\rangle(|0\rangle - |1\rangle)$

$$|S\rangle = \frac{1}{\sqrt{3}}\left(|010\rangle - |011\rangle + |000\rangle\right) = \frac{1}{\sqrt{3}}|0\rangle\left[\underbrace{|10\rangle - |11\rangle + |00\rangle}_{\text{Entangled}}\right]$$

E



acts only on top qubit

$$H|0\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

$$|s_1\rangle = |10\rangle$$

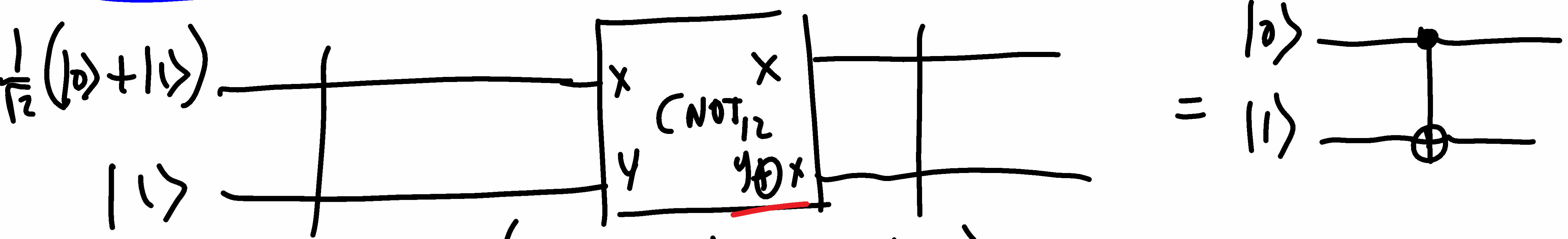
$$X|1\rangle = |0\rangle$$

$$|s_2\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |01\rangle\right)$$

$$|s_3\rangle = \underset{\text{on top qubit}}{X}|s_2\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle X|0\rangle + |0\rangle X|1\rangle\right) = \frac{1}{\sqrt{2}}\left(|01\rangle + |00\rangle\right)$$

Indep Qubits

F Q CIRCUIT THAT CREATES ENTANGLEMENT



$|S_1\rangle = \frac{1}{\sqrt{2}}(|10\rangle + |11\rangle)$

$|S_2\rangle$

$$|S_2\rangle = CNOT_{12}\,|S_1\rangle = \frac{1}{\sqrt{2}}\left(CNOT_{12}|10\rangle + CNOT_{12}|11\rangle\right) =$$

$$= \frac{1}{\sqrt{2}}\left(|10\rangle + |01\rangle\right)$$

Entangled!!

Thu Nov 8 2018

Exercise

Factor qubits as much as possible:

$$\equiv \quad \left(|010\rangle - |011\rangle\right)\frac{1}{\sqrt{2}}$$

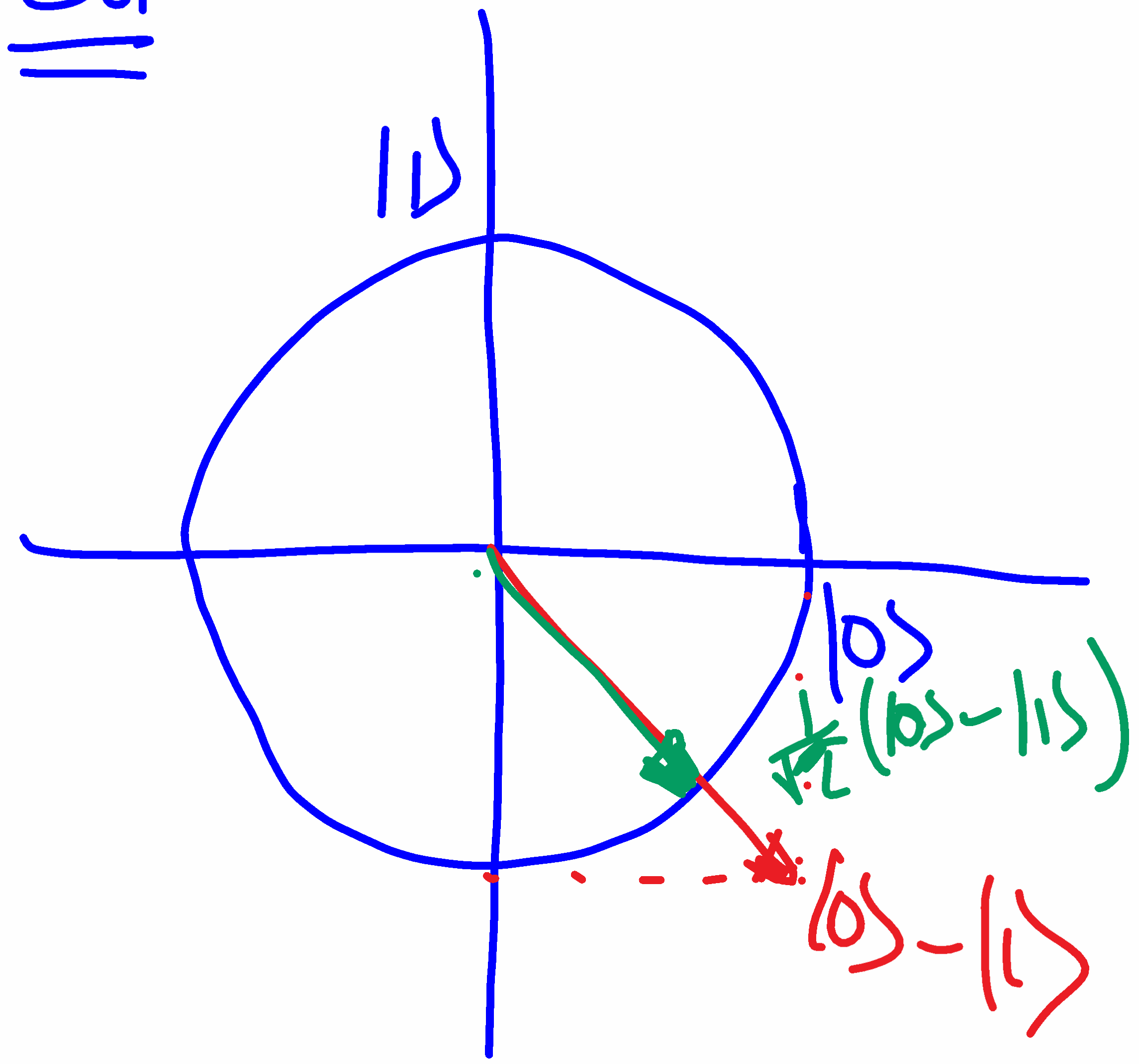
Sol:

$$|0\rangle\left(\frac{|10\rangle - |11\rangle}{\sqrt{2}}\right) = |0\rangle\,|1\rangle\left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right)$$

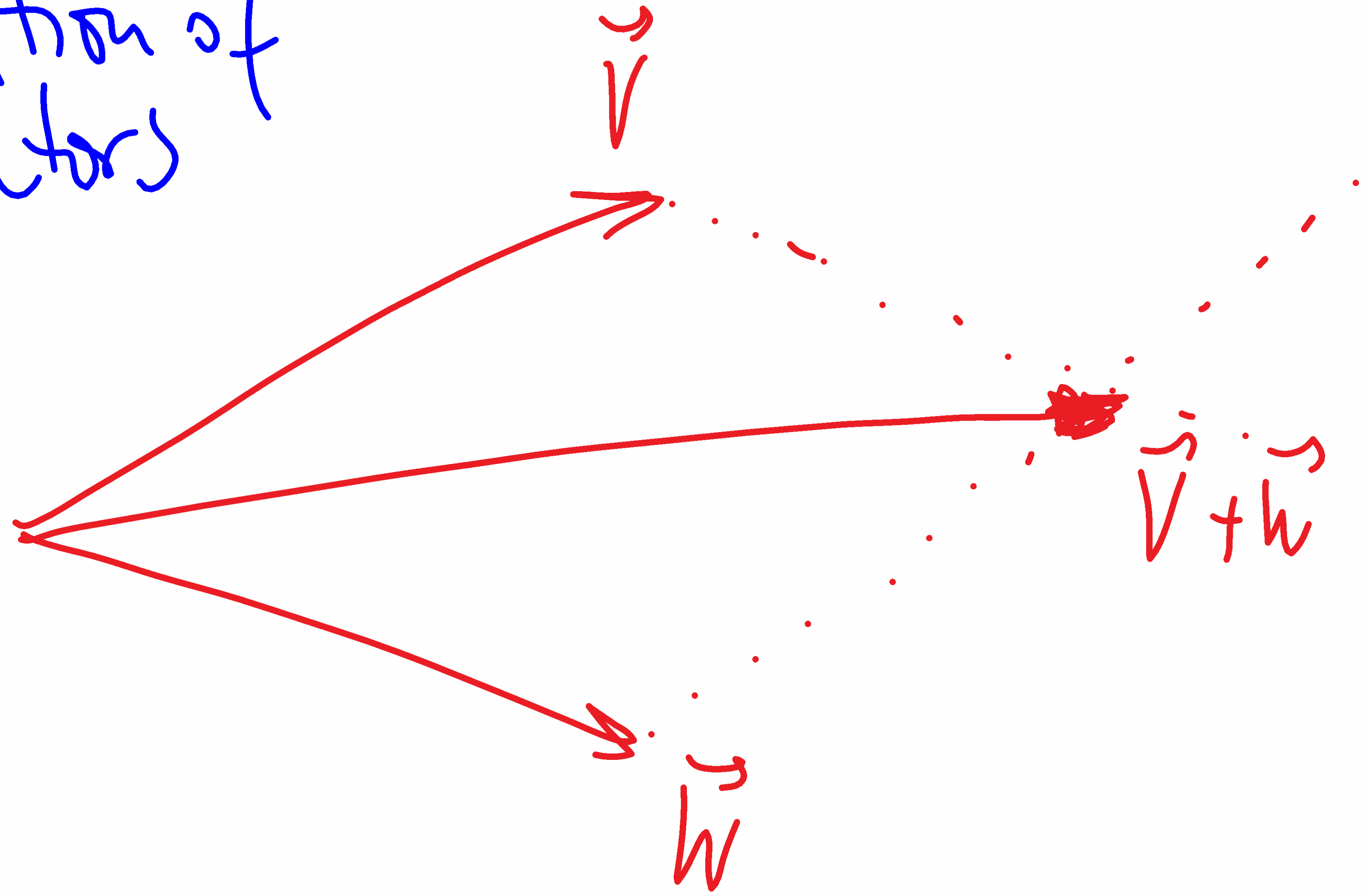
Draw a diagram representing a) $|0\rangle - |1\rangle$

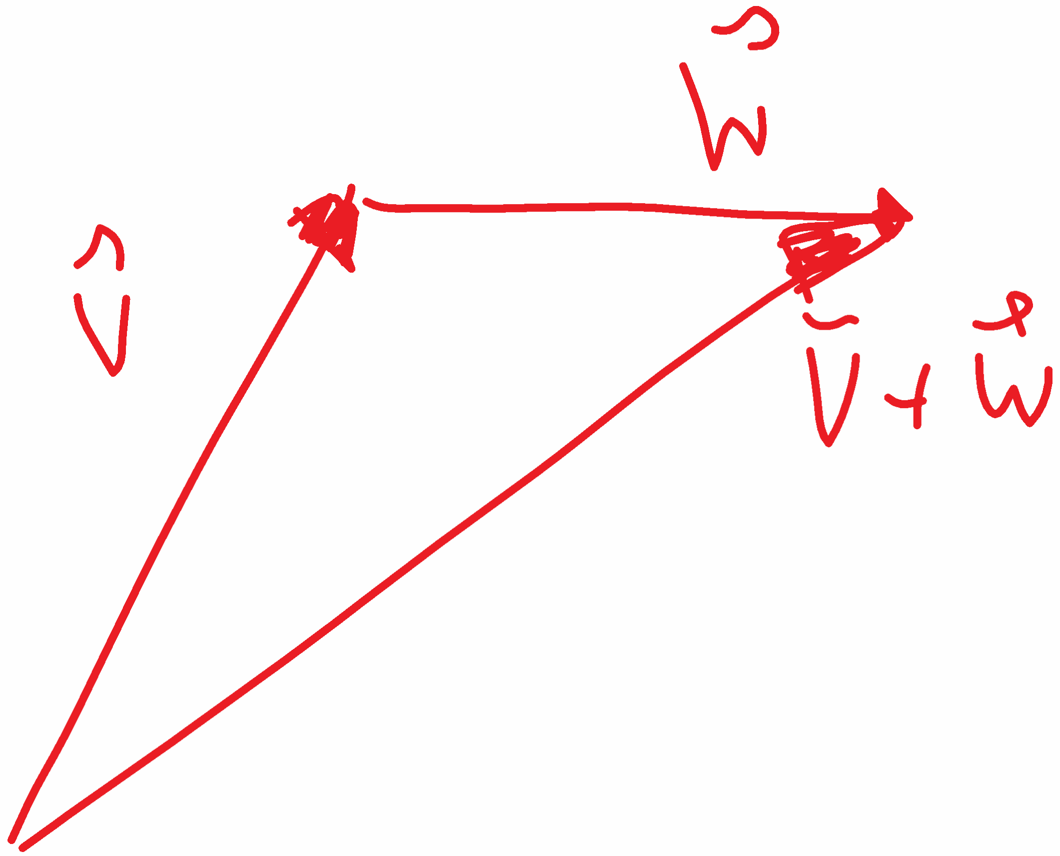
b) $\left(|0\rangle - |1\rangle\right)\frac{1}{\sqrt{2}}$

Sol



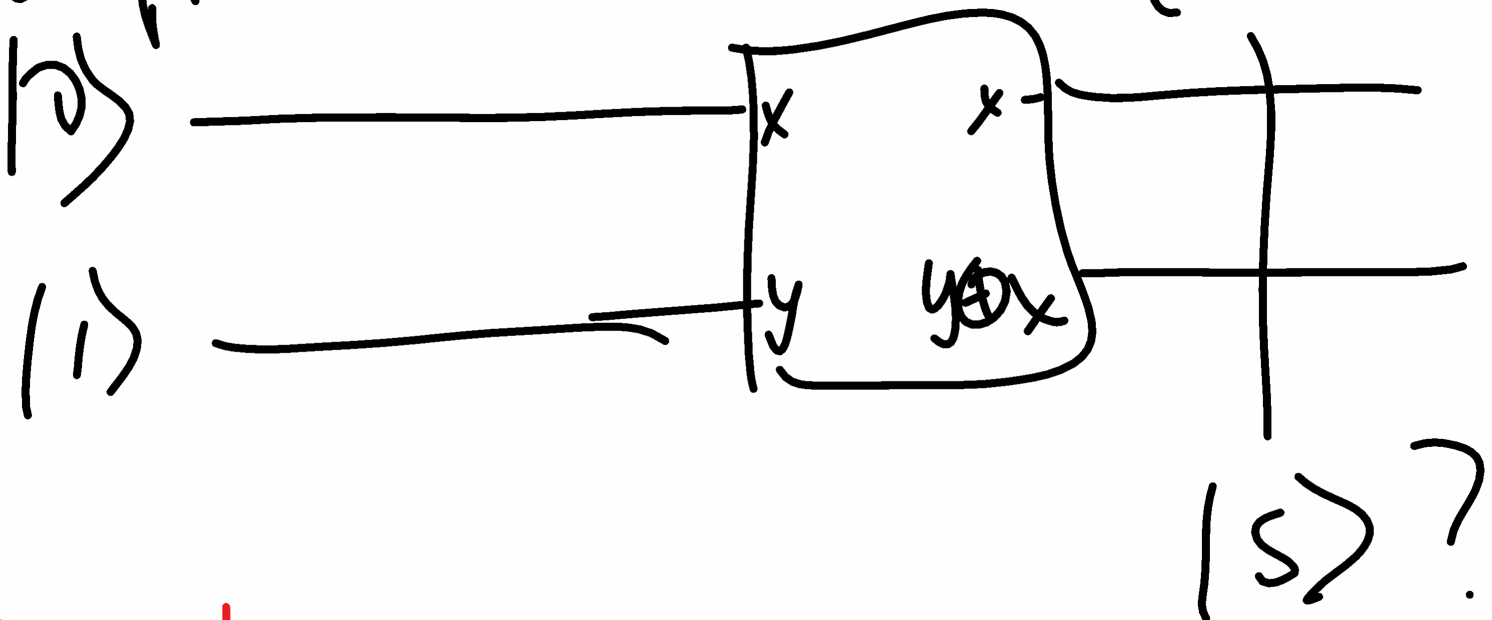
Addition of vectors





(Review additional examples of yesterday)

Example 1



Akin to Truth table

$$|s\rangle = CNOT_{12}\,|10\rangle = |10\rangle$$

$$\text{"}\quad |00\rangle = |00\rangle$$

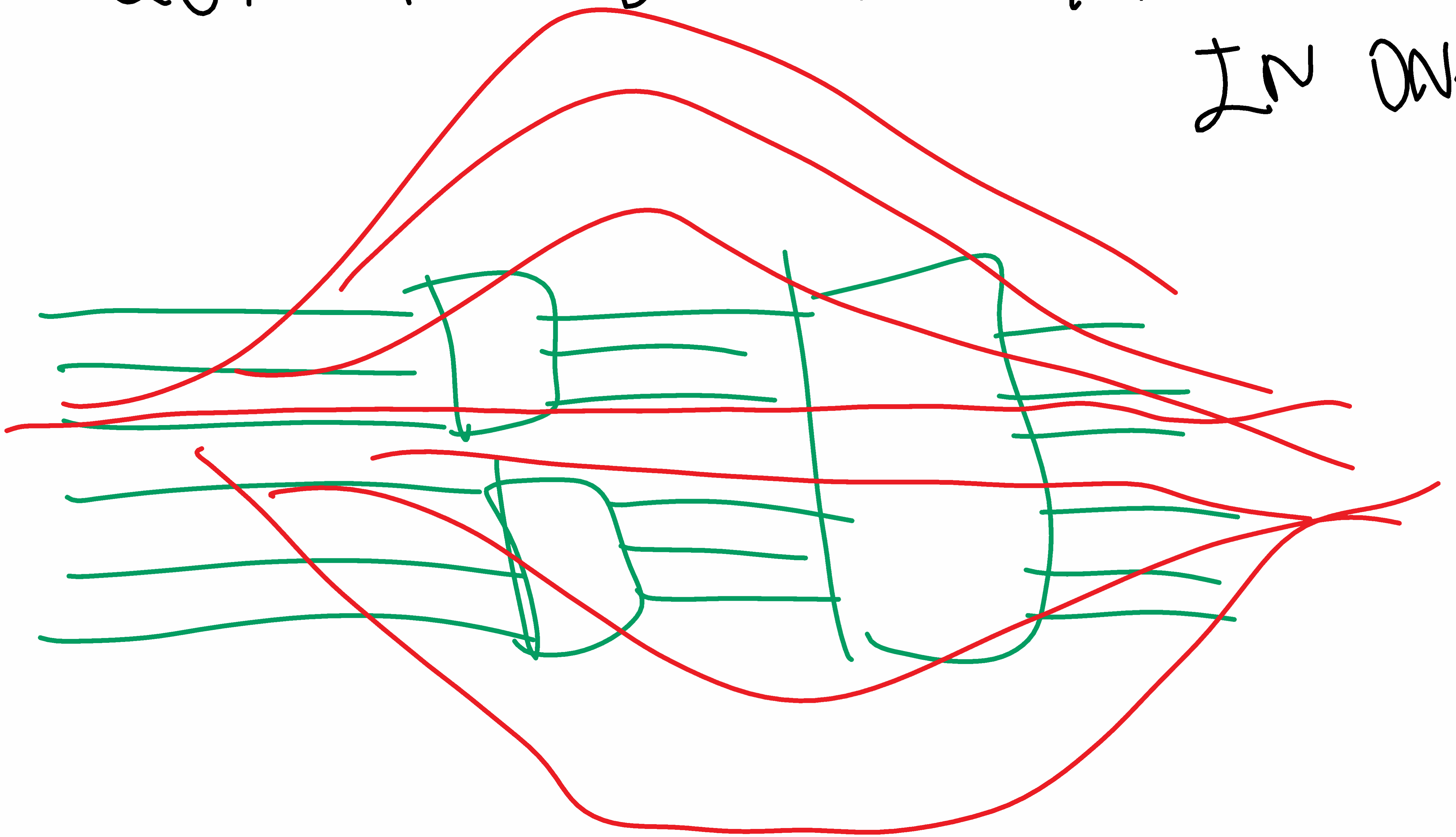
$$\text{"}\quad |01\rangle = |11\rangle$$

$$\text{"}\quad |11\rangle = |01\rangle$$

RULE:

If we want to know what a Quantum Gate does, it's enough to see what it does to each fundamental state of system

IN QUANTUM COMP. EXPLORES ALL CASES IN ONE SINGLE RUN



For instance

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IBM : QUANTUM COMPUTER

Tue 13 Nov 2018

https://quantumexperience.ng.bluemix.net/qx/editor

- MAX 5 Qubits
- Allows simulating the Q Computer
- " Running the Q Comp for real
- Different interfaces for programming
  - GUI
  - QASM
  - Python API

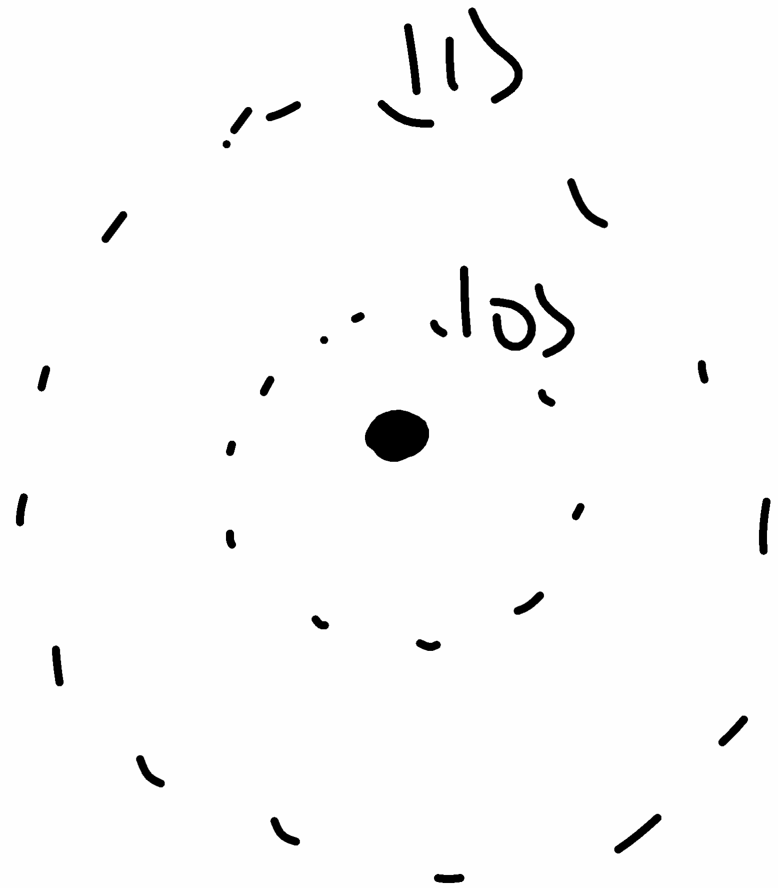
MANY QUANTUM CIRCUIT SIMULATORS

Ex. http://algassert.com/quirk

more: Google

ACTUAL Q COMPUTER is HIGHLY SUSCEPTIBLE TO PERTURBATIONS FROM ENVIRONMENT. Eg. Heat

$\Rightarrow$ Gives rise to two problems



A $|1\rangle \longrightarrow |0\rangle$ "Excited" state gets unstable

The time that $|1\rangle$ stays stable is called T1

$\Rightarrow$ You better do your calculations in LESS THAN T1 seconds

B After a time T2 we loose the precise information of any superposition

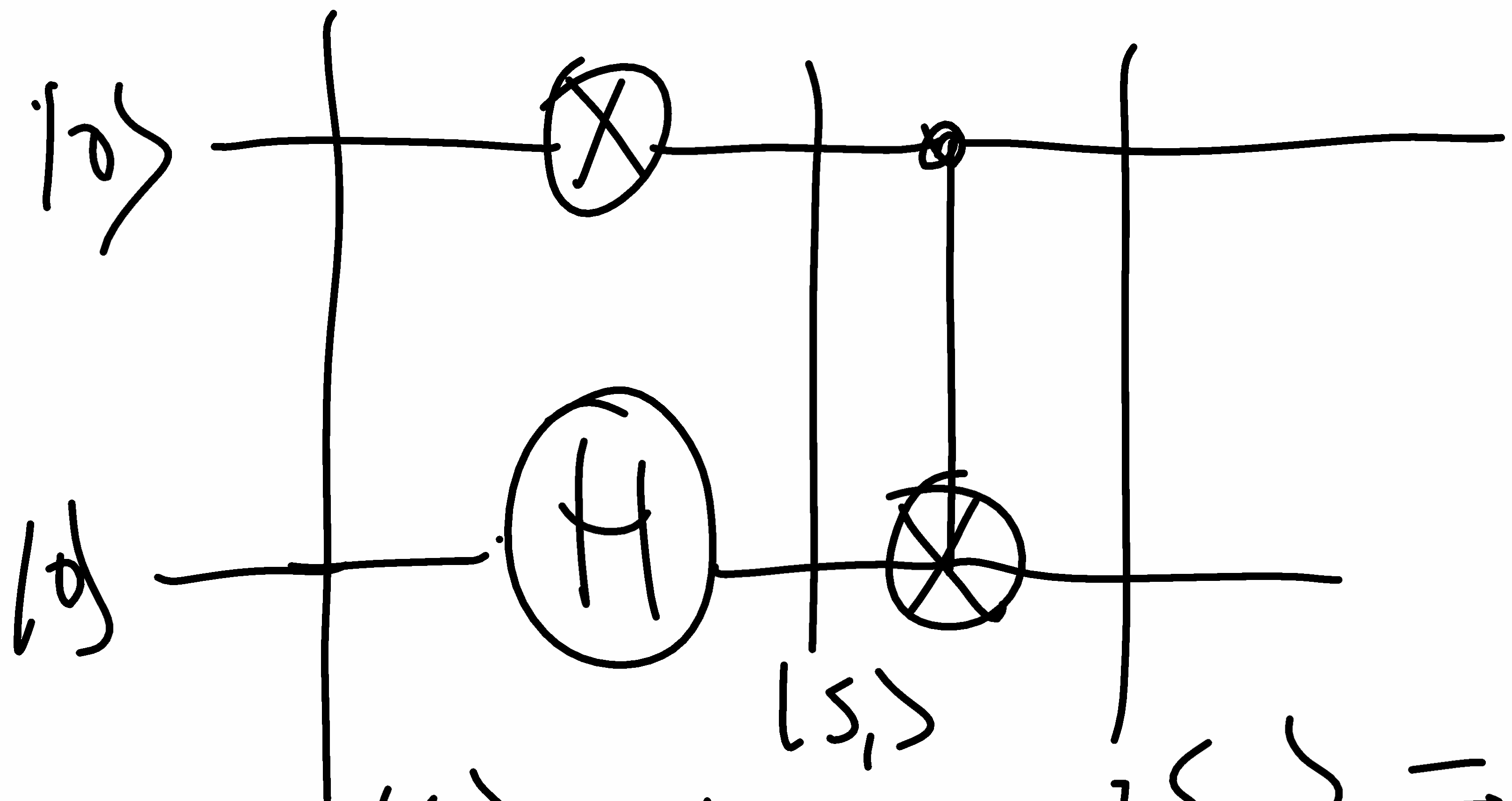
$$\frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right) \xrightarrow[T2]{} a|0\rangle + b|1\rangle$$

?

DUE TO THIS PHYSICAL LIMITATIONS

THE WHOLE SCIENCE OF Q COMP RELIES ON ERROR CORRECTING ALGORITMS

Es. Parity check



$$|S_0\rangle = |00\rangle$$

$$|S_1\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)|1\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle + |11\rangle\right)$$

$$|S_3\rangle = CNOT_{12}\,|S_2\rangle = \frac{1}{\sqrt{2}}\left(\overbrace{CNOT_{12}\,|01\rangle}^{|11\rangle} + \overbrace{CNOT_{12}\,|11\rangle}^{|01\rangle}\right)$$

$$|S_3\rangle = |S_1\rangle$$

ASSIGNMENT 4

1) $|W1\rangle = \left(H|1\rangle\right)\left(H|0\rangle\right) =$

$= \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right) \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$

$= \frac{1}{2}\left(|00\rangle + |01\rangle - |10\rangle - |11\rangle\right)$

$|W2\rangle = \left(ZH|1\rangle\right)\left(XH|0\rangle\right)$