MORE ON EXPONENTIAL FUNCTIONS

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EXAMPLE 2
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How much was invested at 4% compounded semi-annually for 3 years if the final amount was $7500?

Sol Let's write first the function

Say $A(n) \equiv$ Amount of $ at n-half years

$A_0 \equiv$ Initial amount

$$A(n) = A_0 (1+0.04)^n$$

Problem asks for A_0

3 year = 6 "semi-anually"

Information from Problem Statement

They say that $\boxed{A(6) = 7500}$

Hence $A_0 1.04^6 = 7500$

Simplifying $A_0 \cdot 1.27 = 7500$

$1.04^6 \simeq 1.27$

Divide by

$$A_0 = \frac{7500}{1.27} \simeq \boxed{5905.51}$$

Example 3 pg 482

What annual interest was charged if an $800 credit card bill grew to $970.99 in 6 months an the interest was compounded monthly?

Sol: let's summarize the "story": We spent $800 w/ our credit card and missed to pay it by the deadline. The bank then started charging an interest on that debt. The interest gets compounded monthly. After 6 months we owe the bank $920.99

Write the formula $A(n) = A_0(1+i)^n$

$A_0 = 800$ The statement says that

$$A(6) = 920.99$$

$$800(1+i)^6 = 920.99$$

Isolate the parentheses by dividing by 800

$$(1+i)^6 = \frac{920.99}{800} \approx 1.15$$

Now raise to power $\frac{1}{6}$ $\longrightarrow$ $\left[(1+i)^6\right]^{\frac{1}{6}} = (1.15)^{\frac{1}{6}}$

$$\left[(1+i)^6\right]^{\frac{1}{6}} = 1.15^{1/6}$$

$$(1+i)^{\frac{6}{6}} = 1.15^{1/6}$$

$$1+i = 1.15^{1/6}$$

$$i = 1.15^{1/6} - 1 = \sqrt[6]{1.15} - 1 \simeq 0.0236$$

Hence $\boxed{i = 2.36\%}$

Example 4
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Approximately, how long would it take for a $1500 investment to double if it earns a 10%/yr interest compounded semi-annually?

Sol: Write formula

$$A(n) = P(1+i)^n$$

$P = 1500$

$i = \frac{0.1}{2} = .05$

$\Rightarrow A(n) = 1500 \cdot 1.05^n$

One way approx:

$1.05^2 = 1.1025$

$1.05^{14} = 1.98$

$1.05^{15} = 2.08$

We want to find when $1.05^n = 2$