Term 4 Review

Thu 30 May 2019

- Powers
- Exponential Func
- Trigonometric ratios
- Sinusoidal / Periodic Func

Powers

$$b^n \equiv 1 \cdot \underbrace{b \cdot \ldots \cdot b}_{n}$$

Ex: $3^4 = 1 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

Rules

- $b^n \cdot b^m = b^{n+m}$ Ex: $5^7 \cdot 5^2 = 5^{7+2} = 5^9$
- $\frac{b^n}{b^m} = b^{n-m}$ Ex: $\frac{5^7}{5^2} = 5^{7-2} = 5^5$
- $(b^n)^m = b^{n \cdot m}$ Ex: $(5^2)^7 = 5^{14}$

- $(a \cdot b)^n = a^n \cdot b^n$ Ex: $(2 \cdot 3)^2 = 2^2 \cdot 3^2 = 4 \cdot 9 = 36$

- $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ Ex: $\left(\frac{5}{3}\right)^7 = \frac{5^7}{3^7}$

- $b^{-n} = \frac{1}{b^n}$ Ex: $3^2 = \frac{1}{3^{-2}}$

  $5^{-7} = \frac{1}{5^7}$

- $b^{\frac{1}{n}} = \sqrt[n]{b}$ Ex: $8^{\frac{1}{3}} = \sqrt[3]{8} = 2$

EXPONENTIAL FUNC

Ex: $f(x) = 2^x$ Ex: $y = x^2$ is not exponential fun

In general $f(x) = a\, b^{\frac{x-e}{d}} + c$

Ex: $y = 7 \cdot 3^{-\frac{x}{2}}$ | $a = 7$ $e = 0$ $d = -2$
$b = 3$ $c = 0$

$y = 5 \cdot 2^x + 3$ | $a = 5$ $c = 3$ $d = 1$
$b = 2$ $e = 0$

- DECAY PROCESSES

Iodine 135 half-life of $t_{1/2} = 30$ days

If initially we have 10^3 kg, how much remains after 1 yr? (Assume 1 yr = 365 days)

Sol:

Amount of radioactive material with the time

$$A(d) = 100\left(\frac{1}{2}\right)^{d/30}$$

(gives the percentage after d days)

After 1 yr $A(365) \simeq 100\left(\frac{1}{2}\right)^{\frac{365}{30}} \simeq 100\left(\frac{1}{2}\right)^{12.17}$

$\simeq 100 \cdot 0.000217 = 0.0217\%$

Mass after 1 yr is

$$10^3 \cdot \frac{0.0217}{100} = 0.217\,\text{kg}$$

$$= \underline{217\,\text{gr}}$$

· Investments

$$A(n) = P\cdot(1+i)^n$$

$A \equiv$ Amount

$n \equiv$ number of compounding periods

$P \equiv$ initial amount, aka., the principal

$i \equiv$ interest rate per compounding period

Ex: You plan an investment at $7\frac{3}{4}\%/a$ that is compounded semi-annually! If you want to have \$800 after 5 yrs, how much do you need to invest initially?

Sol: $A(n) = P(1+i)^n$

$n = 5 \cdot 2 = 10$

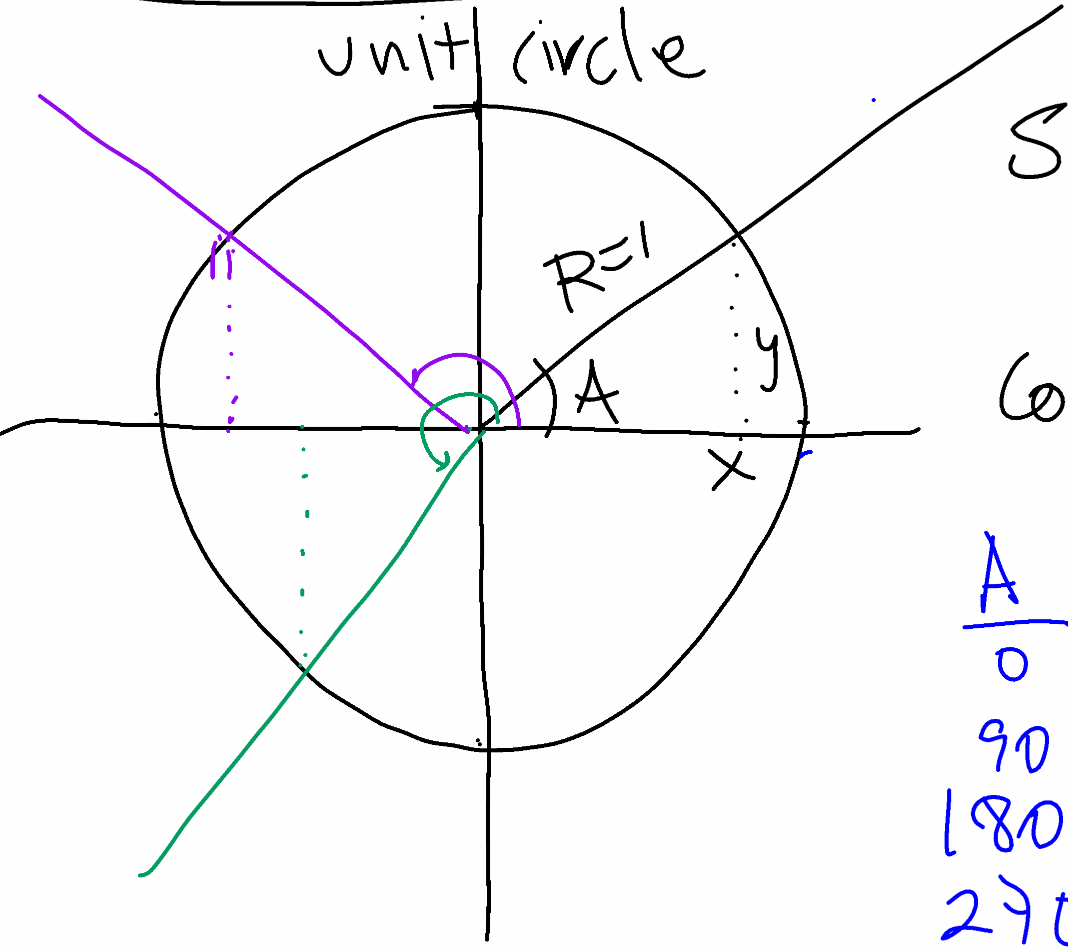
$A(10) = 800$

$i = \frac{7+\frac{3}{4}}{200} = \frac{\frac{28+3}{4}}{200} = \frac{31}{800} \approx 0.04 = 4\%$

$$800 = P(1+0.04)^{10} = P(1.04)^{10} \simeq P \cdot 1.48$$

$$P = \frac{800}{1.48} \simeq \boxed{540.45\ \$}$$

TRIGONOMETRIC RATIOS



$$\sin A = \frac{y}{R} = y$$

$$\cos A = \frac{x}{R} = x$$

$$\tan A = \frac{\sin A}{\cos A} = \frac{y}{x}$$

A	Sin A	Cos A
0	0	1
90	1	0
180	0	-1
270	-1	0
360	0	1

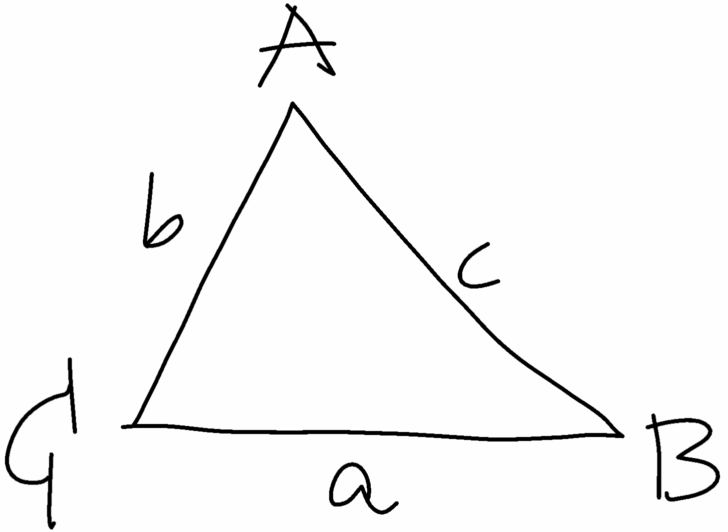
Fundamental Trig Relation

$$\sin^2 A + \cos^2 A = 1$$

It comes from Pythagoras Theo

$$y^2 + x^2 = R^2$$

SOLVING TRIANGLES



SINE LAW

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

or

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

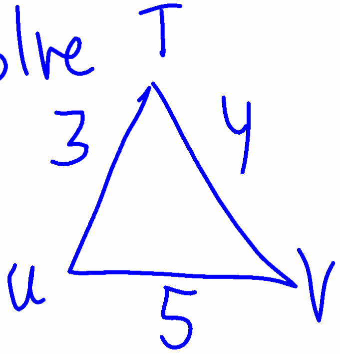
COSINE LAW

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Ex: Solve

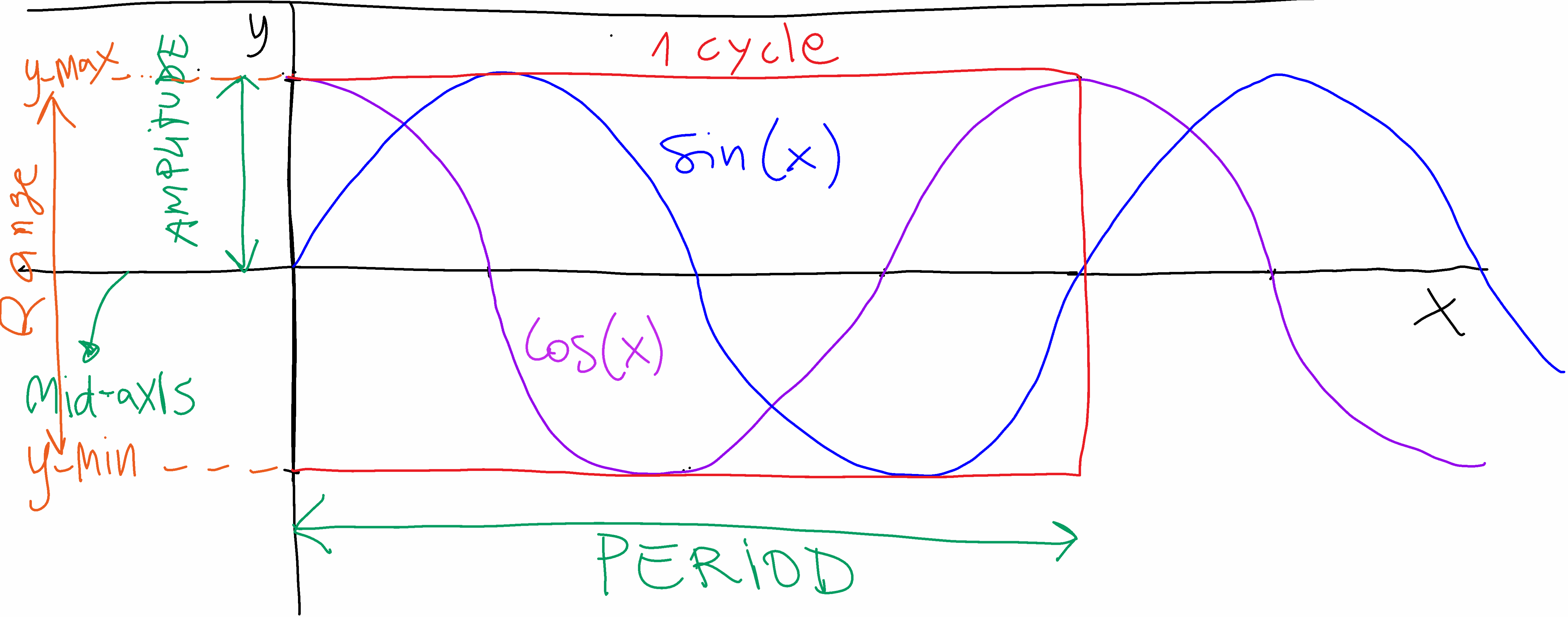


$$5^2 = 3^2 + 4^2 - 2\cdot 3\cdot 4\cdot \cos T$$

$$4^2 = 3^2 + 5^2 - 2\cdot 3\cdot 5\cos U$$

$$V = 180 - T - U$$

SINUSOIDAL & PERIODIC FUNCTIONS



Equation of the mid-axis: $y = \frac{y_{max} + y_{min}}{2}$

Amplitude: $A = \frac{y_{max} - y_{min}}{2}$

<u>Example</u>: $f(t) = 0.25 \sin(180 \cdot t)$

We get the max value of $f(t)$ when $\sin = 1$ $\Rightarrow f_{max} = 0.25$

We get the min value of $f(t)$ when $\sin = -1$ $\Rightarrow f_{min} = -0.25$

Hence, $y_{mid\text{-}axis} = \frac{0.25 - 0.25}{2} = 0$; $A = \frac{0.25 + 0.25}{2} = 0.25$

Range of $f(t) = 0.25 \sin(180t)$: $[-0.25, 0.25]$

PERIOD: Is given by the value of T for which the sin makes a full cycle, i.e., when $180 \cdot T = 360$

$$\boxed{T = \frac{360}{180} = 2}$$

Problem: Find the value of t for which $f(t) = 0.25 \cdot \sin(180 \cdot t)$ reaches its max

<u>Sol</u> That value of t will be the one for which $\sin(180 \cdot t)$ reaches its maximum.

But the sine reaches its max when the angle is 90.

Hence, the sought for value is $180 \cdot t = 90 \Rightarrow \boxed{t = \frac{1}{2}}$