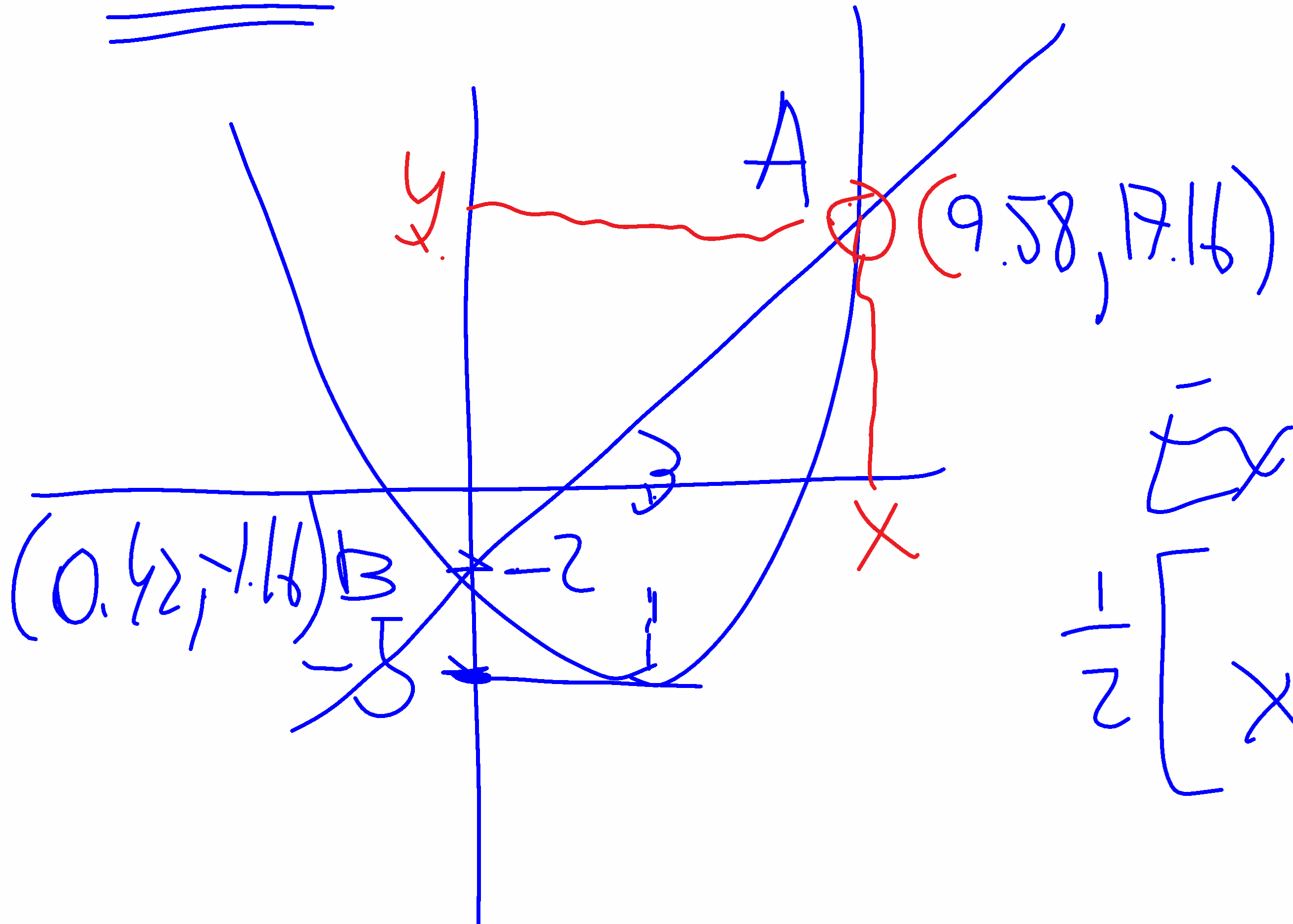
Q11: Determine how many points of intersection there are between the parabola $y=\frac{1}{2}(x-3)^2-5$ and the line $y=2x-2$. Find the intersection points.

Sol



At the x where they intersect it is

$$\boxed{\frac{1}{2}(x-3)^2-5=2x-2}$$

Expand

$$\frac{1}{2}\left[x^2-2\cdot 3\cdot x+3^2\right]-5=2x-2$$

$$\frac{1}{2}\left[x^2-6x+9\right]-5=2x-2$$

$$\frac{x^2}{2}-3x+\frac{9}{2}-5=2x-2$$

$$\frac{x^2}{2}-3x-2x+\frac{9}{2}-5+2=0$$

$$ax^2+bx+c=0$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$\frac{x^2}{2}-5x+\frac{3}{2}=0 \qquad a=\frac{1}{2} \quad b=-5 \quad c=\frac{3}{2}$$

$$\frac{9}{2}-5+2=\frac{9-10+4}{2}=\frac{3}{2}$$

$$x=\frac{5\pm\sqrt{5^2-\cancel{4}\cdot\frac{1}{\cancel{2}}\cdot\frac{3}{\cancel{2}}}}{\cancel{2}\cdot\frac{1}{\cancel{2}}}=5\pm\sqrt{25-3}=$$

$$=5\pm\sqrt{21}=\begin{cases} + & 9.58 \\ - & 0.42 \end{cases}$$

The y-coordinates of the intersection points can be obtained by substituting these x-values in the equation of the line

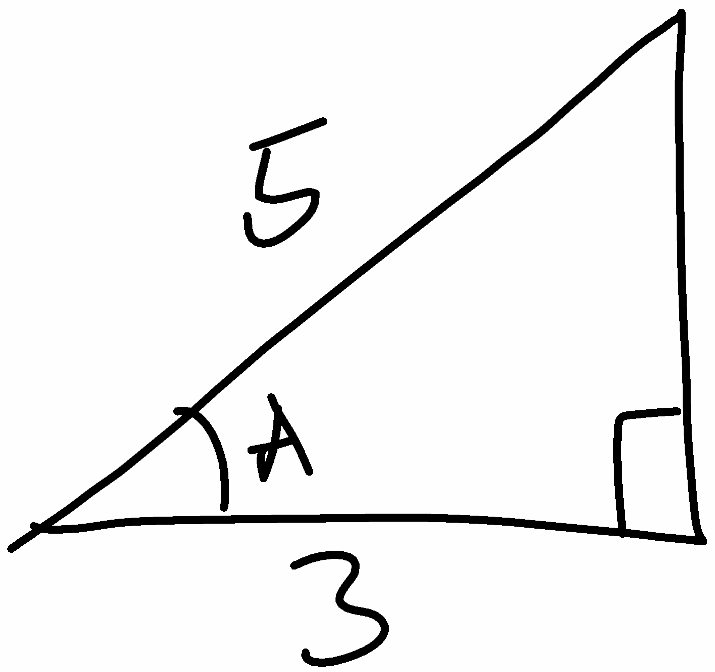
A: $y = 2 \cdot 9.58 - 2 = 17.16$

$$\boxed{A = (9.58\,,\,17.16)}$$

B: $y = 2 \cdot 0.42 - 2 = -1.16$

$$\boxed{B = (0.42\,,\,-1.16)}$$

Q12: Determine the basic trigonometric ratios for A



Sol We have to determine the values of $\sin A$, $\cos A$ & $\tan A$

$$\boxed{\cos A = \frac{3}{5}}$$

Fundamental Trigonometric Relation

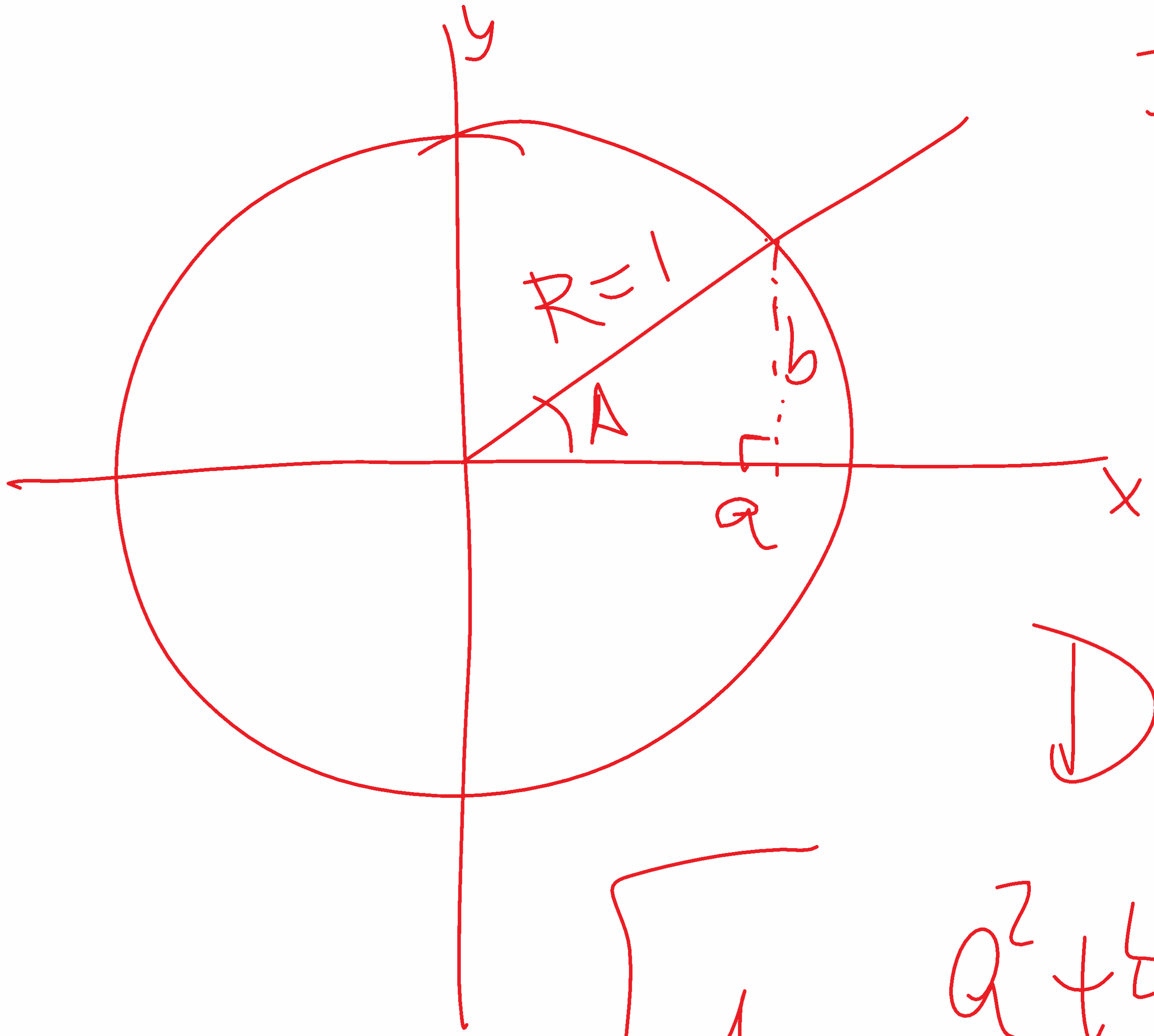
$$\sin^2 A + \cos^2 A = 1$$

$$\sin^2 A + \left(\frac{3}{5}\right)^2 = 1 \rightarrow \sin A = \sqrt{1 - \frac{9}{25}}$$

$$\boxed{\sin A = \sqrt{\frac{16}{25}} = \frac{4}{5}}$$

$$\boxed{\tan A = \frac{\sin A}{\cos A} = \frac{4/5}{3/5} = \frac{4}{3}}$$

Fund. Trigono. Relation



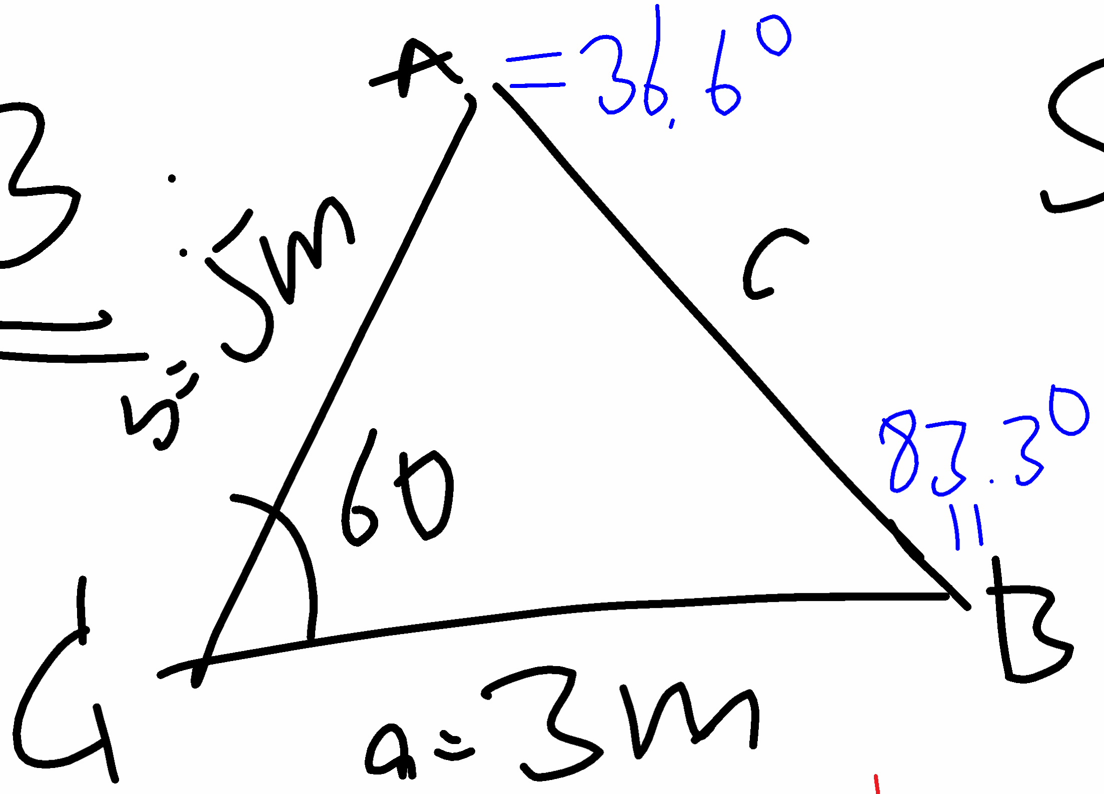
Pythagoras Theorem

$$R^2 = a^2 + b^2$$

Divide both sides by R^2

$$\boxed{1 = \frac{a^2+b^2}{R^2} = \frac{a^2}{R^2} + \frac{b^2}{R^2} = \left(\frac{a}{R}\right)^2 + \left(\frac{b}{R}\right)^2 = \cos^2 A + \sin^2 A}$$

Q13



Solve: SAS ⇒ Cosine law

Cosine law

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$c^2 = a^2 + b^2 - 2ab \cdot \cos 60$$

$$c^2 = 9 + 25 - 30 \cdot \frac{1}{2} = 9 + 25 - 15$$

$$c^2 = 19 \Rightarrow \boxed{c = \sqrt{19} \simeq 4.36\,m}$$

Sine law

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

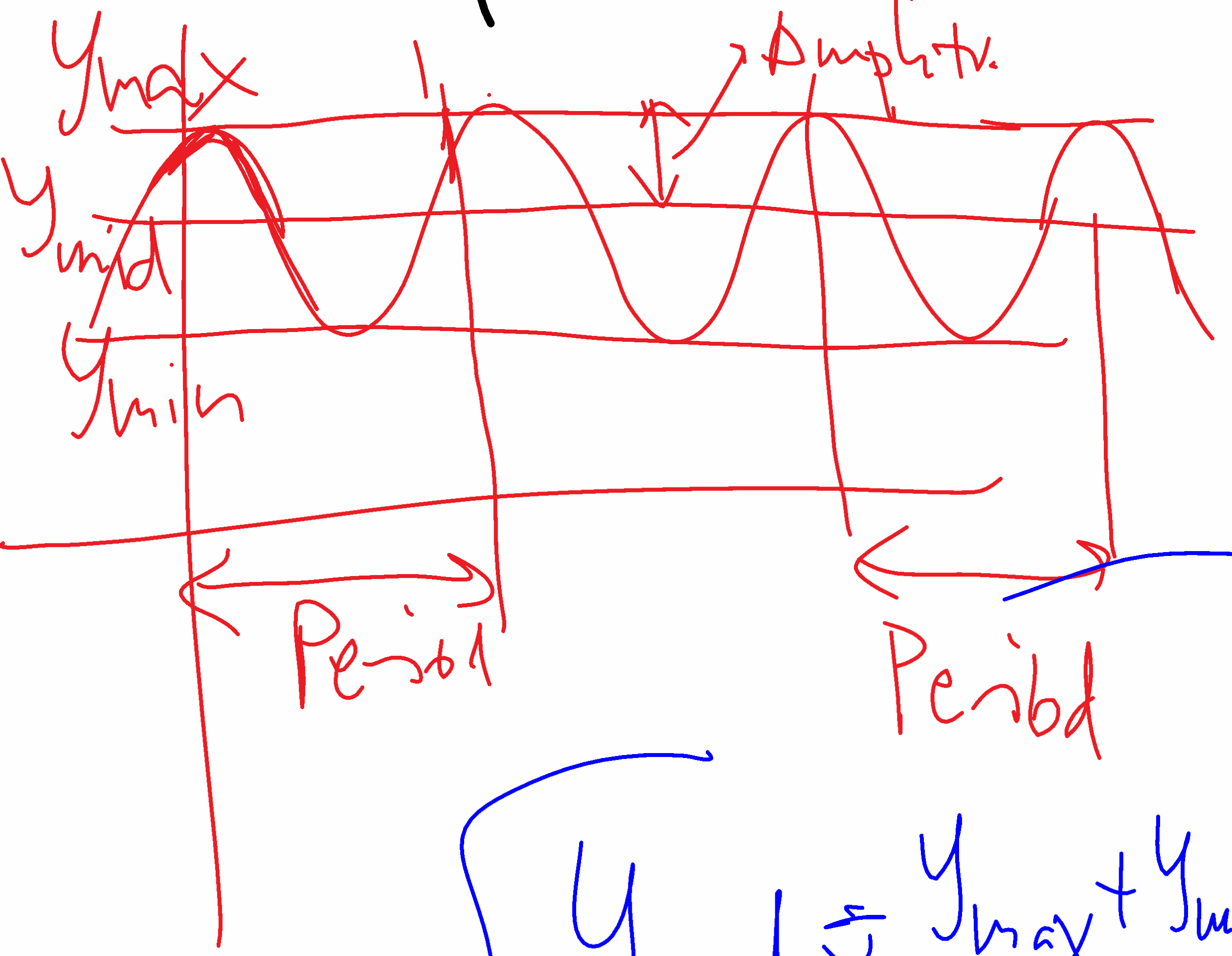
$$\frac{\sin A}{a} = \frac{\sin C}{c} \rightarrow \frac{\sin A}{3} = \frac{\sin 60}{4.36} \rightarrow \sin A = \frac{3 \cdot \sin 60}{4.36} = \simeq 0.5959$$

$$\boxed{A \simeq 36.6^\circ}$$

$$\frac{\sin B}{5} = \frac{\sin 60}{4.36} \rightarrow \sin B = \frac{5\sin 60}{4.36} \Rightarrow \boxed{B \simeq 83.3^\circ}$$

Q14 Determine max. value, min value, mid-point axis, and the period of the following sinusoidal function

$$f(x) = -3\cos(2x) + 7$$



Sol

$$\boxed{y_{min} = -3 \cdot 1 + 7 = 4}$$

$$\boxed{y_{max} = -3(-1) + 7 = 10}$$

$$\boxed{y_{mid} = \frac{y_{max} + y_{min}}{2} = 7}$$

$$\boxed{\text{Amplitude} = \frac{y_{max} - y_{min}}{2} = 3}$$

Period:

$2x = 360 \rightarrow \boxed{P = 180}$

Q15 A radioactive material has a half-life of 130 days. What's the percentage of material remaining after 2 yrs?

$t_{1/2} = 130$ days

Sol: $2\,yrs = 2 \cdot 365$ Days $= 730$ days

Decay formula

$$A(t) = 100\left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}$$

$t_{1/2}$ = "half-life"

$$A(730) = 100\left(\frac{1}{2}\right)^{\frac{730}{130}} = 100\left(\frac{1}{2}\right)^{\frac{73}{13}} \simeq 100 \cdot 0.02$$

$$\boxed{A(730) \simeq 2\%}$$

Q16: The bank gives you a loan of \$30.000 at an annual interest of 4.5%/a compounded semi-annually. If you pay back in full after 5yrs, how much do you pay?

$$A(n) = P(1+i)^n$$

$A \equiv$ Amount

$i \equiv$ interest in one period

$n \equiv$ number of compounding periods

$P \equiv$ principal, aka, initial amount

Sol: $n = 5\,\text{yrs}\ \dfrac{2\,\text{periods}}{\text{yr}} = 10$

$i = \dfrac{4.5\%}{1\,\text{yr}}\ \dfrac{1\,\text{yr}}{2\,\text{periods}} = \dfrac{2.25\%}{\text{period}} = 0.0225$

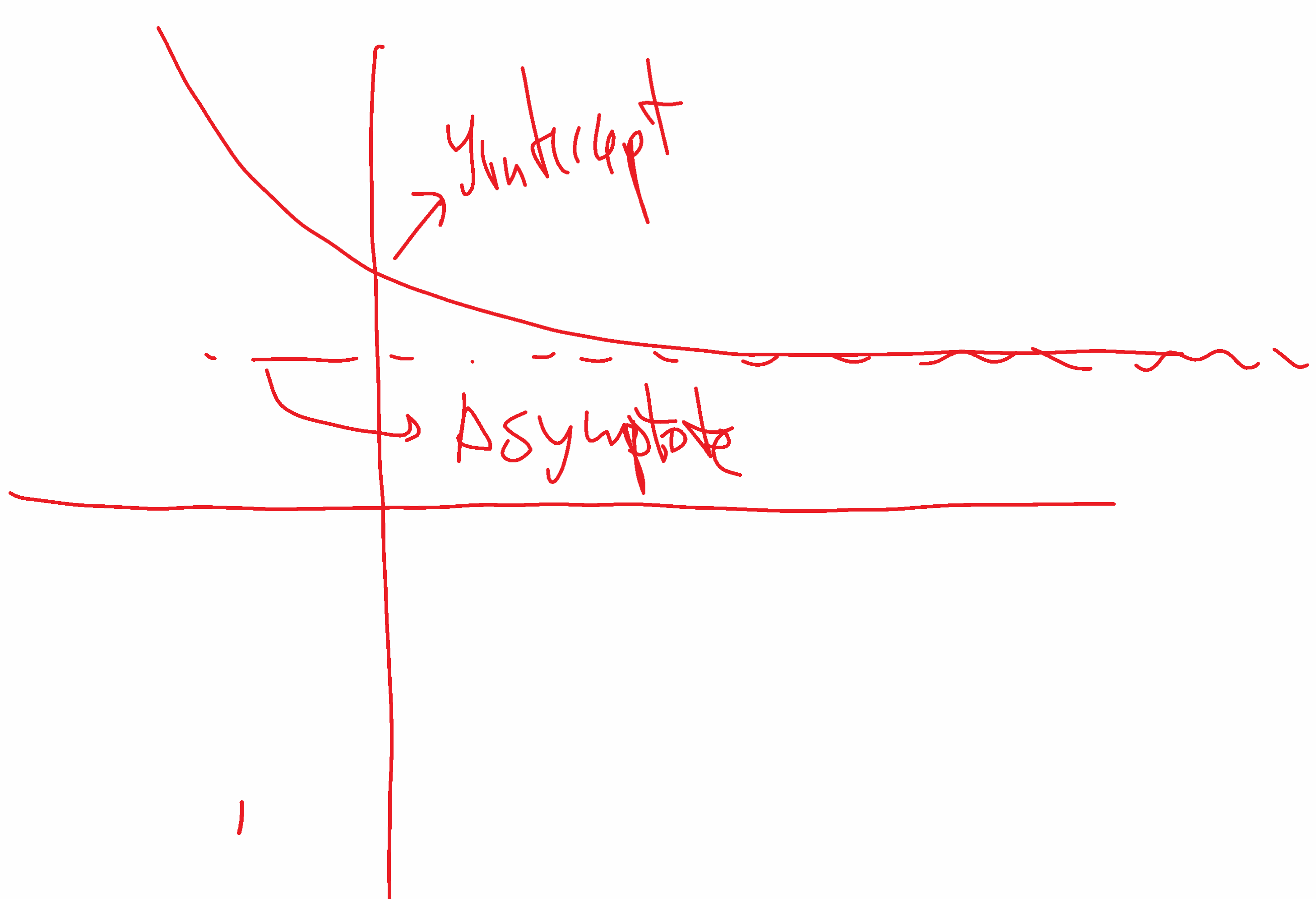
$P = 30000$

Hence, $\boxed{A(10)} = 30000(1+0.0225)^{10} \simeq$

$\simeq 30000 \cdot 1.2492 = \boxed{37476.10\$}$

Q18: Determine the horizontal asymptote & the y-intercept of the function $f(x) = 3\left(\frac{1}{2}\right)^{\frac{t}{5}} - 1$

Exponential function



Sol: y-intercept $= f(0) = 3\left(\frac{1}{2}\right)^{0} - 1 = 2$

$y_{asymp} = -1$