The Dragon Academy
G11 Functions and Applications
Term 3
Assignment 1

February 15, 2019

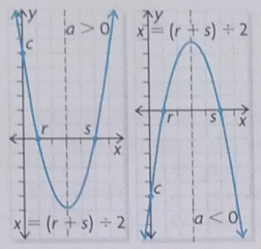
In Summary

Key Ideas

- Some quadratic functions in standard form, $f(x) = ax^2 + bx + c$, can be expressed in factored form as $f(x) = a(x - r)(x - s)$ by factoring. The two forms of the quadratic function are equivalent.
- All quadratic functions in factored form can be expressed in standard form by expanding. The two forms of the quadratic function are equivalent.

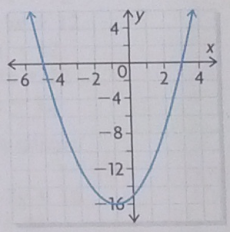
Need to Know

- Both the standard and factored forms provide useful information for graphing the parabola.
  - When a quadratic function is expressed in factored form, $f(x) = a(x - r)(x - s)$, r and s are the x-intercepts, or zeros, of the function. The axis of symmetry is the vertical line that runs through the midpoint of the zeros and is defined by $x = (r + s) \div 2$. This value is also the x-coordinate of the vertex.
  - When a quadratic function is expressed in standard form, $f(x) = ax^2 + bx + c$, c is the y-intercept of the function.
  - The value of a in both factored and standard forms determines the direction that the parabola opens. If $a > 0$, the parabola open up; if $a < 0$, the parabola opens down.



CHECK Your Understanding

1. The parabola shown is congruent to $y = x^2$.
   a) What are the zeros of the function?
   b) Write the equation in factored form.



2. Express each quadratic function in factored form. Then determine the zeros, the equation of the axis of symmetry, and the coordinates of the vertex.
   a) $f(x) = 2x^2 + 12x$
   b) $f(x) = x^2 - 7x + 12$
   c) $f(x) = -x^2 + 100$
   d) $f(x) = 2x^2 + 5x - 3$

3. Express each quadratic function in standard form. Determine the y-intercept.
   a) $f(x) = 3x(x - 4)$
   b) $f(x) = (x - 5)(x + 7)$
   c) $f(x) = 2(x - 4)(3x + 2)$
   d) $f(x) = (3x - 4)(2x + 5)$

Communication | *Tip*

Two parabolas that are congruent have exactly the same size and shape.

PRACTISING

4. For each quadratic function, determine the zeros, the equation of the axis of symmetry, and the coordinates of the vertex without graphing.

a) $g(x) = 2x(x + 6)$
b) $g(x) = (x - 8)(x + 4)$
c) $g(x) = (x - 10)(2 - x)$
d) $g(x) = (2x + 5)(9. - 2x)$
e) $g(x) = (2x + 3)(x - 2)$
f) $g(x) = (5 - x)(5 + x)$

5. Express each function in factored form. Then determine the zeros, the equation of the axis of symmetry, and the coordinates of the vertex without graphing.

a) $g(x) = 3x^2 - 6x$
b) $g(x) = x^2 + 10x + 21$
c) $g(x) = x^2 - x - 6$
d) $g(x) = 3x^2 + 12x - 15$
e) $g(x) = 2x^2 - 13x - 7$
f) $g(x) = -6x^2 + 24$

6. Match the factored form on the left with the correct standard form on the right. How did you decide on your answer?

a) $y = (2x + 3)(x - 4)$
b) $y = (4 - 3x)(x + 3)$
c) $y = (3x - 4)(x - 3)$
d) $y = (3 - 4x)(4 - x)$
e) $y = (x + 3)(3x - 4)$
i) $y = 4x^2 - 19x + 12$
ii) $y = -3x^2 - 5x + 12$
iii) $y = 2x^2 - 5x - 12$
iv) $y = 3x^2 - 13x + 12$
v) $y = 3x^2 + 5x - 12$

7. Determine the maximum or minimum value for each quadratic function.

a) $f(x) = (7 - x)(x + 2)$
b) $f(x) = (x + 5)(x - 9)$
c) $f(x) = (2x + 3)(8 - x)$
d) $g(x) = x^2 + 7x + 10$
e) $g(x) = -x^2 + 25$
f) $g(x) = 4x^2 + 4x - 3$

8. K Graph each quadratic function by hand by determining the zeros, vertex, axis of symmetry, and y-intercept.

a) $g(x) = (2x - 1)(x - 4)$
b) $f(x) = (3x - 1)(2x - 5)$
c) $f(x) = x^2 - x - 20$
d) $g(x) = 2x^2 + 2x - 12$
e) $f(x) = -x^2 - 2x + 24$
f) $f(x) = -4x^2 - 16x + 33$

9. a) When a quadratic function is in standard form, what information about the graph can be easily determined? Provide an example.

b) When a quadratic function is in factored form, what information about the graph can be easily determined? Provide an example.

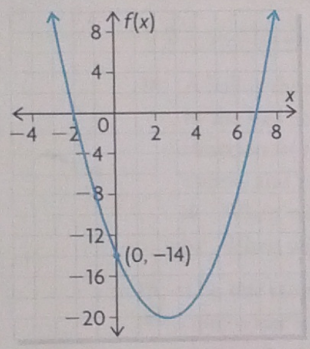
10. Graph each function, and complete the table.

	Factored Form	Standard Form	Axis of Symmetry	Zeros	*y*-intercept	Vertex	Maximum or Minimum Value
a)	$R(x) = (40 - x)(10 + x)$	$R(x) = -x^2 + 30x + 400$					
b)	$f(x) = (x - 4)(x + 2)$						
c)		$g(x) = -x^2 + 2x + 8$					
d)	$p(x) = (3 - x)(x + 1)$						
e)		$j(x) = 4x^2 - 121$					

11. A The height of water, $h(t)$, in metres, from a garden hose is given by the function $h(t) = -5t^2 + 15t$, where t is time in seconds. Express the function in factored form, and then use the zeros to determine the maximum height reached by the water.

12. For each graph, write the equation in both factored and standard forms.

a)



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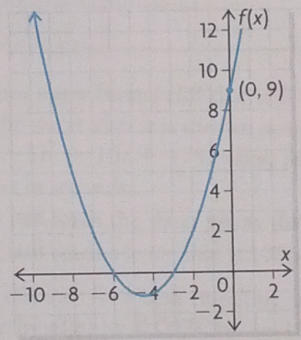

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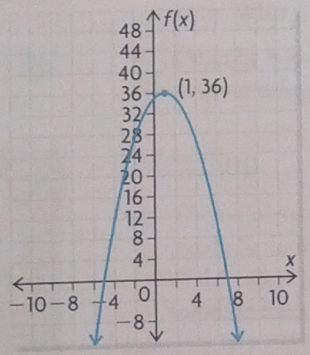

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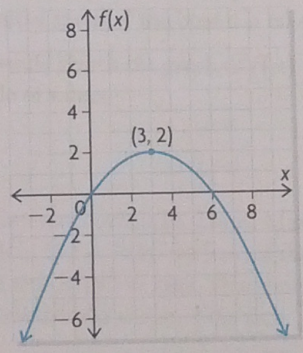
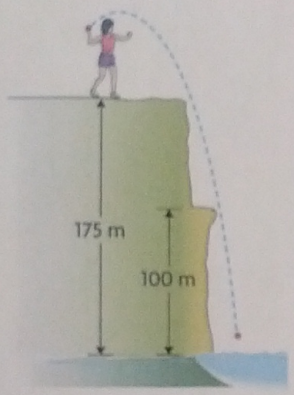


13. Complete the table.

	Zeros	Axis of Symmetry	Maximum or Minimum Value	Vertex	Function in Factored Form	Function in Standard Form
a)	2 and 8		6			
b)	−7 and −2		−2			
c)	−1 and 9		5			
d)	−8 and 0		−5			



14. A ball is thrown into water from a cliff that is 175 m high. The height of the ball above the water after it is thrown is modelled by the function $h(t) = -5t^2 + 10t + 175$, where $h(t)$ is the height in metres and t is time in seconds.
    a) When will the ball reach the water below the cliff?
    b) When will the ball reach a ledge that is 100 m above the water?

15. T The safe stopping distance for a boat travelling at a constant speed in calm water is given by $d(v) = 0.002(2v^2 + 10v + 3000)$, where $d(v)$ is the distance in metres and v is the speed in kilometres per hour. What is the initial speed of the boat if it takes 30 m to stop?

16. C Describe how you would sketch the graph of $f(x) = 2x^2 - 4x - 30$ without using a table of values.

Extending

17. The stainless-steel Gateway Arch in St. Louis, Missouri, is almost parabolic in shape. It is 192 m from the base of the left leg to the base of the right leg. The arch is 192 m high. Determine a function, in standard form, that models the shape of the arch.

18. A model rocket is shot straight up into the air. The table shows its height, $h(t)$, at time t. Determine a function, in factored form, that estimates the height of the rocket at any given time.

Time (s)	0	1	2	3	4	5	6
Height (m)	0.0	25.1	40.4	45.9	41.6	27.5	3.6

19. The path of a shot put is given by $h(d) = 0.0502(d^2 - 20.7d - 26.28)$, where $h(d)$ is the height and d is the horizontal distance, both in metres.
    a) Rewrite the relation in the form $h(d) = a(d - r)(d - s)$, where r and s are the zeros of the relation.
    b) What is the significance of r and s in this question?