# The Dragon Academy
# G11 Functions and Applications
# Term 3
# Assignment 5
# **Due date: Thu. Apr. 11 2019**

April 5, 2019

This assignment is the first one on the current unit of exponential function. See the slides of Fri. 5 Apr. 2019 in our course page.

It comprises the following exercises from the book:

1. Page 390, Exercises: $5, 6$.
2. Page 400, Exercises: $5, 8$.
3. Page 408, Exercises: $5, 6$ (yes, again the same numbers).
4. Page 415, Exercises: $1, 2$.

Pictures of the pages for those lacking the book have been attached below in this document.

## PRACTICE

Evaluate questions 4 to 8 without using a calculator.

**4.** Evaluate.

a) $8^3$ c) $5^6$ e) $4^5$

b) $11^4$ d) $19^2$ f) $2^{10}$

**5.** Evaluate.

a) $(-5)^2$ c) $(-2)^3$ e) $(-10)^4$

b) $-5^2$ d) $-2^3$ f) $-10^4$

**6.** Evaluate.

a) $(-3^3)^3$ c) $[(-3)^4]^2$ e) $(-3^3)^2$

b) $[(-3)^3]^3$ d) $(-3^4)^2$ f) $(-3^2)^3$

**7.** Evaluate.

a) $3^2 - 4^2$ c) $(1 + 7^2)^2$ e) $5^2 \times (-2)^3$

b) $10^2 - 15^1 + 5^2$ d) $(6^2 - 4^2)^2$ f) $8^2 \div (-4)^3$

**8.** Evaluate.

a) $\sqrt{25} + \sqrt{16}$ b) $\dfrac{\sqrt{100}}{\sqrt{25}}$ c) $\sqrt{\sqrt{81}}$

**9.** Determine the exponent that makes each of the following true.

a) $2^x = 16$ c) $3^y = 27$ e) $(-2)^n = -8$

b) $17^m = 17$ d) $4^x = 64$ f) $5^c = 125$

**10.** Evaluate.

a) $\dfrac{4}{7} - \dfrac{3}{4}$ c) $\dfrac{2}{3}\left(\dfrac{5}{4}\right)$ e) $\dfrac{4}{9}\left(\dfrac{9}{5} - \dfrac{3}{10}\right)$

b) $\dfrac{7}{9} \div \dfrac{4}{5}$ d) $\dfrac{2}{3} + \left(\dfrac{5}{4}\right)$ f) $\left(\dfrac{9}{10}\right)\dfrac{3}{7} \div \dfrac{3}{14}$

**11.** Determine the first and second finite differences for each set of data. State whether each set represents a linear or a quadratic relationship.

a)

| x | y |
|---|---|
| −3 | 14 |
| −2 | 10 |
| −1 | 6 |
| 0 | 2 |
| 1 | −2 |
| 2 | −6 |
| 3 | −10 |

b)

| x | y |
|---|---|
| −3 | 11.5 |
| −2 | 6.5 |
| −1 | 3.5 |
| 0 | 2.5 |
| 1 | 3.5 |
| 2 | 6.5 |
| 3 | 11.5 |

c)

| x | y |
|---|---|
| −6 | 15 |
| −4 | −3 |
| −2 | −13 |
| 0 | −15 |
| 2 | −9 |
| 4 | 5 |
| 6 | 27 |

**Study Aid**

- For help, see Essential Skills Appendix.

| Question | Appendix |
|---|---|
| 5, 6, 7, and 9 | A-3 |
| 10 | A-2 |
| 11 | A-12 |

## PRACTISING

**4.** Evaluate.

K

a) $10^{-2}$

b) $(-4)^{-2}$

c) $\left(\frac{1}{2}\right)^{-5}$

d) $\left(\frac{1}{7}\right)^{-3}$

e) $\frac{1}{(-9)^2}$

f) $(-5)^0$

**5.** Simplify. Write each expression as a single power with a positive exponent.

a) $9^7 \times 9^{-3}$

b) $6^{-3} \div 6^{-5}$

c) $8^6 \div 8^{-5}$

d) $17^{-4} \div 17^{-6}$

e) $(-3)^{-8} \times (-3)^5$

f) $(-4)^{-5} \times (-4)^5$

**6.** Simplify. Write each expression as a single power with a positive exponent.

a) $2^4(2^2) \div 2^{-6}$

b) $-5 \times (-5^4)^{-3}$

c) $\frac{(-12^3)^{-1}}{(-12)^7}$

d) $\left(\frac{3^4}{3^6}\right)^{-1}$

e) $\frac{9^4(9^3)}{9^{12}}$

f) $((7^2)^{-3})^{-4}$

**7.** Simplify. Write each expression as a single power with a positive exponent.

a) $\frac{11^{-2}(11^3)}{(11^{-2})^4}$

b) $\left(\frac{9^{-2}}{(9^2)^2}\right)^2$

c) $\left(\frac{4^{-3}}{4^{-2}}\right)^{-3}$

d) $\left(\frac{10}{10^{-3}}\right)^2\left(\frac{10^5}{10^7}\right)$

e) $\frac{(-8^{-1})(-8^{-3})}{(-8^{-2})^3}$

f) $\left(\frac{(5^3)^2}{5(5^6)}\right)^{-1}$

**8.** Simplify, then evaluate each expression. Leave answers as fractions or integers.

a) $13^3 \times 13^{-4}$

b) $\frac{3^{-2}}{3^{-6}}$

c) $\left(\frac{10^{-3}}{10^{-5}}\right)^2$

d) $6^{-2}(6^{-2})^{-1}$

e) $\frac{-2(-2^{-3})}{(-2)^4}$

f) $\left(\frac{5^{-2}}{5}\right)^{-1}$

**9.** Evaluate. Leave answers as fractions or integers.

a) $3^{-2} - 9^{-1}$

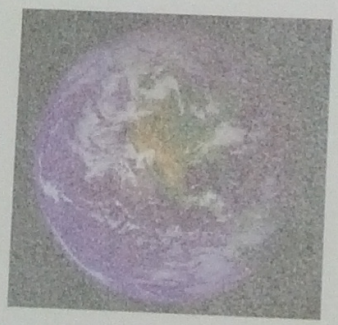
b) $4^{-2} + 3^0 - 2^{-3}$

c) $8^{-2} + (4^{-1})^2$

d) $\left(\frac{1}{2}\right)^{-1} + \left(\frac{1}{3}\right)^{-1}$

e) $12(4^0 - 3^{-2})$

f) $\frac{4^2}{2^5}$



**10.** A Scientific notation can be used to represent very large and very small numbers. The diameter of Earth is about $1.276 \times 10^7$ m, while the diameter of a plant cell is about $1.276 \times 10^{-5}$ m. Explain why negative exponents are used in scientific notation to represent very small numbers.

## PRACTISING

**4.** C Create three examples to help a classmate learn about the following relationships:

a) the result of multiplying powers with the same base

b) the result of dividing powers with the same base

c) the result of raising a power to an exponent

**5.** K Simplify. Write as a single power.

a) $3(3^5) \div 3^3$

b) $10^9 \div (10^3 \times 10^2)$

c) $(7^8 \div 7^5)(7^2)$

d) $\dfrac{(6^2)(6^{11})}{6^8}$

e) $\dfrac{9^{12}}{9(9^{10})}$

f) $\dfrac{(8^7)(8^3)}{8^6(8^2)}$

**6.** Simplify. Write as a single power.

a) $(2^5)^3 \times 2^3$

b) $5^9 \div (5^3)^2$

c) $(7^8)(7^5)^2$

d) $\dfrac{(8^2)^5}{8^8}$

e) $\dfrac{10(10^9)}{(10^2)^3}$

f) $\dfrac{(4^7)^3}{4^9(4^{11})}$

**7.** Simplify. Write as a single power.

a) $10(10^5)(10^3) \div (10^3)^2$

b) $\dfrac{(8^8)(8^3)^3}{8^3(8^{11})}$

c) $\left(\dfrac{13(13^{12})}{13^7}\right)^2$

d) $\dfrac{(5^4)^2(5^5)^2}{5^2(5^{13})}$

**8.** Simplify, then evaluate without using a calculator.

a) $\left(\dfrac{4}{3}\right)\left(\dfrac{4}{3}\right)^2$

b) $\left(\dfrac{1}{9}\right)^4 \div \left(\dfrac{1}{9}\right)^2$

c) $\left(\left(\dfrac{2}{5}\right)^2\right)^2$

d) $\left(\dfrac{5}{4}\right)^5\left(\dfrac{5}{4}\right)^3 \div \left(\dfrac{5}{4}\right)^6$

**9.** Simplify.

a) $x^4(x^2)^2$

b) $\dfrac{(m^5)^2}{m^8}$

c) $(y(y^6))^3$

d) $((a^2)^2)^2$

e) $a^2\,a^2\,a^2$

f) $\dfrac{b(b^5)b^4}{b^5}$

**10.** Write each power in simplified form.

a) $4^3$ as a base 2 power

b) $9^5$ as a base 3 power

c) $27^5$ as a base 3 power

d) $(-8)^4$ as a base $-2$ power

e) $\left(\dfrac{1}{4}\right)^3$ as a base $\dfrac{1}{2}$ power

f) $\left(\dfrac{1}{25}\right)^4$ as a base $\dfrac{1}{5}$ power

## In Summary

### Key Ideas

- A power with a rational exponent is equivalent to a radical. The rational exponent $\frac{1}{n}$ indicates the $n$th root of the base. If $n > 1$ and $n \in \mathbb{N}$, then $b^{\frac{1}{n}} = \sqrt[n]{b}$, where $b \neq 0$.
- If $m \neq 1$ and if $m$ and $n$ are both positive integers, then $b^{\frac{m}{n}} = \left(\sqrt[n]{b}\right)^m = \sqrt[n]{b^m}$, where $b \neq 0$.

### Need to Know

- The exponent laws that apply to powers with integer exponents also apply to powers with rational exponents.
- The power button on a scientific calculator can be used to evaluate rational exponents.
- Some roots of negative numbers cannot be determined. For example, $-16$ does not have a real-number square root, since $(-4)^2 = (-4) \times (-4) = +16$. Odd roots can have negative bases, but even ones cannot.
- Since radicals can be written as powers with rational exponents:
  - Their products are equivalent to the products of powers. This means that $\sqrt{a} \times \sqrt{b} \times \sqrt{c} = \sqrt{a \times b \times c}$, because $a^{\frac{1}{2}} \times b^{\frac{1}{2}} \times c^{\frac{1}{2}} = (abc)^{\frac{1}{2}}$, where $a$, $b$, and $c > 0$.
  - Their quotients are equivalent to the quotient of powers. This means that $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$, because $\frac{a^{\frac{1}{2}}}{b^{\frac{1}{2}}} = \left(\frac{a}{b}\right)^{\frac{1}{2}}$, where $a$, $b$, and $c > 0$.

## CHECK Your Understanding

1. Write in radical form. Then evaluate without using a calculator.
   a) $49^{\frac{1}{2}}$
   b) $(-125)^{\frac{1}{3}}$
   c) $81^{\frac{1}{4}}$
   d) $100^{\frac{1}{2}}$
   e) $16^{0.25}$
   f) $-(144)^{0.5}$

2. Write in exponent form. Then evaluate.
   a) $\sqrt[10]{1024}$
   b) $\sqrt[5]{1024}$
   c) $\sqrt[3]{27^4}$
   d) $\left(\sqrt[3]{-216}\right)^5$
   e) $\sqrt[4]{16}$
   f) $(\sqrt{25})^{-1}$

3. Use your calculator to evaluate each expression to the nearest hundredth.
   a) $6^{\frac{2}{5}}$
   b) $0.0625^{\frac{1}{4}}$
   c) $\sqrt[15]{4421}$
   d) $144^{0.25}$
   e) $10^{\frac{-2}{3}}$
   f) $200^{-0.4}$