The Dragon Academy
G11 Functions and Applications
Term 4
Homework 3
Due date: Fri. 24 2019

May 28, 2019

1 Problems

This is a set of exercises to review what we have seen today in class.

Remember:

- A ***periodic*** function is a function that **repeats** itself after a given interval
- The ***period*** of a periodic function is the distance along the x-axis that takes for that function to **repeat** itself

Example: Imagine an airport where there is a plain departing every 3 minutes.

- The Departures is a function that repeats itself. Hence it is a ***periodic*** function.
- The ***period*** of the function Departures is 3min.

Question: What's the period of the function $f(x) = \sin(x)$? Ans.: The function $\sin(x)$ repeats itself every $360°$ degrees, hence its period is $360°$.

List of Problems From our book: Page 373, Exercises: 1, 4. From our book: Page 376, Exercises: 19. Below you have a picture of the statements.

Key Idea

- The graph of the function $f(x) = a \sin(x - c°) + d$ looks periodic in the same way the graph of $f(x) = \sin x$ does. The differences are only in the placement of the graph and how stretched or compressed it is.

Need to Know

- If $f(x) = a \sin x$, the value of a has the following effect on the function $f(x) = \sin x$:
  - When $a > 1$, the function is stretched vertically by the factor a.
  - When $0 < a < 1$, the function is compressed vertically by the factor a.
  - When $a < -1$, the function is stretched vertically by the factor a and reflected across the x-axis.
  - When $-1 < a < 0$, the function is compressed vertically by the factor a and reflected across the x-axis.
- A function of the form $f(x) = a \sin(x - c°) + d$ results from applying transformations to the graph of $f(x) = \sin x$ in the following order:
  1. Horizontal translations: determined by the value of c
  2. Stretches/compressions: determined by the value of a;
     Reflections: necessary only when $a < 0$
  3. Vertical translations: determined by the value of d

CHECK Your Understanding

1. State the transformations that are applied to $f(x) = \sin x$.
   a) $f(x) = 3 \sin x$
   b) $f(x) = -2 \sin x$
   c) $f(x) = 0.1 \sin x$
   d) $f(x) = -\frac{1}{3} \sin x$

2. The graph of $f(x) = \sin x$ has been compressed by a factor of 5 and reflected in the x-axis. Write the new equation.

3. a) Sketch the sinusoidal function $f(x) = 4 \sin x$. Verify your answer with graphing technology.
   b) State the period, amplitude, and the equation of the axis.
   c) State the domain and range.

PRACTISING

4. State whether a vertical stretch or a compression results for each of the following.
   a) $f(x) = 2 \sin x$
   b) $f(x) = 0.5 \sin x$
   c) $f(x) = -\frac{1}{4} \sin x$
   d) $f(x) = -4 \sin x$
   e) $f(x) = \frac{1}{3} \sin x$
   f) $f(x) = 10 \sin x$

18. Explain how you can graph a sinusoidal function that has undergone more than one transformation. C

Extending

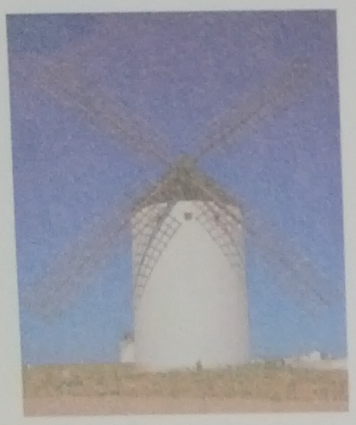
19. State the period of each function.

a) $f(x) = \sin\left(\frac{1}{2}x\right)$

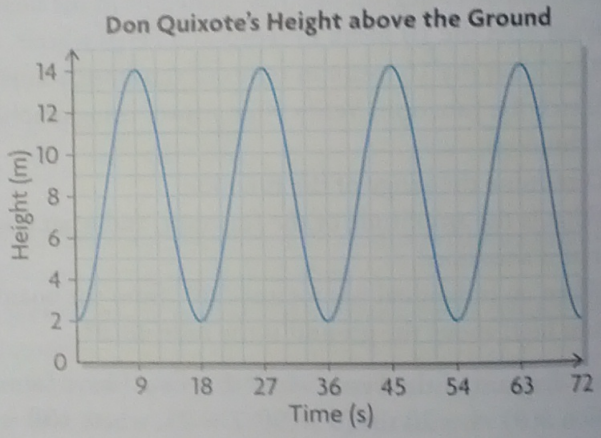
b) $f(x) = \sin\left(\frac{1}{4}x\right)$

c) $f(x) = \sin(2x)$

d) $f(x) = \sin(10x)$



20. Don Quixote, a fictional character in a Spanish novel, attacked windmills, thinking they were giants. At one point, he got snagged by one of the blades and was hoisted into the air. The graph shows his height above the ground in terms of time.



a) What is the equation of the axis of the function, and what does it represent in this situation?

b) What is the amplitude of the function, and what does it represent in this situation?

c) What is the period of the function, and what does it represent in this situation?

d) What transformation would generate the period for this graph?

e) Determine the equation of the sinusoidal function.

f) If the wind speed decreased, how would that affect the graph of the function?

g) If the axle for the windmill were 1 m higher, how would that affect the graph of the function?