The Dragon Academy
G12 Mathematics of Data Management
Term 3
Test 1

February 25, 2020

The first 6 questions amount to 90% of the total mark of this test, each of them weighing the same.

1 Name:

2 Questions

1. (KtiCa) Natasha tosses four coins, one after the other.

   (a) In how many different orders could heads or tails occur?
   (b) Draw a tree diagram to illustrate all the possible resutls
   (c) Explain how your tree diagram corresponds to your calculation in part 1.

2. (KtiCa) How many four-digit numbers can we form with the digits $1, 2, 3, 4, 5, 6, 7$ if no digit is repeated? How many of these four-digit numbers are odd? How many even?

3. (ktiCA) Determine the number of partitions of a set of 5 elements into two blocks of 2 and 3 elements.

4. (Ktica) Rewrite each of the following as a single combination

   (a) $\binom{10}{7} + \binom{10}{8}$
   (b) $\binom{23}{15} + \binom{22}{14}$

5. (kTIcA) Using the binomial therorem

   (a) expand $(8x - 3)^5$
   (b) Calculate the value of the following function for $x = 3/4$, $y = 1/4$, $z = 1$:

$$f(x, y, z) = (x + y)^{20} \cdot \sum_{k=0}^{k=10} \binom{10}{k} \left[\frac{z}{(x + y)^2} \right]^k$$

6. (ktiCA) A binary relation between two sets A and B is defined as a list (set) of *ordered* pairs (a, b) where the first element belongs to A, $a \in A$, and the second to B, $b \in B$. Example: Be $A = \{1, 2, 3\}$, $B = \{5, 7, 11\}$. The following are possible binary relations between A and B: $R_0 == $, $R_1 = (2, 7)$, $R_2 = (1, 9), (1, 5), (2, 5)$, etc.

   (a) What is the number of binary relations with 3 ordered pairs between the given A and B?
   (b) How many distinct binary operations are there between the given A and B? Use the binomial theorem to give an exact answer without a calculator (that is, from your writing it should be clear how we can calculate the answer without calculator. Write your final answer as a power if you don't know its integer value).

(c) The same question when A has 4 elements and B 5.

7. (ktiCA) A logic function $C = f(A.B)$ is a relation such that 1) for each combination of the arguments (here A and B) it outputs one and onely one value C and 2) all values, arguments and images are binary values, say 0 or 1. In Logic, each such functions is called a (logic) operator. If it has just 1 arguments, it's called a *uniray* operator; if 2 arguments, it's called a *binary* operator.

(a) How many different binary operators are there?

(b) In the gneeral case, how many logic operators are there in n variables? $(C = f(A_1, A_2, \ldots, A_n))$

(c) How would your previous two answers change if we would use a logic with 3 truth values, namely `True, False, Maybe`?