The Dragon Academy
G12 Mathematics of Data Management
Term 3
Assignment 2
Due date: Wed. Feb. 12th 2020

February 7, 2020

All exercices weigh the same towards the final mark on this assignment.
See the attached pictures for the statements of each problem.
List of problems:

1. **Page 260:** 1-10
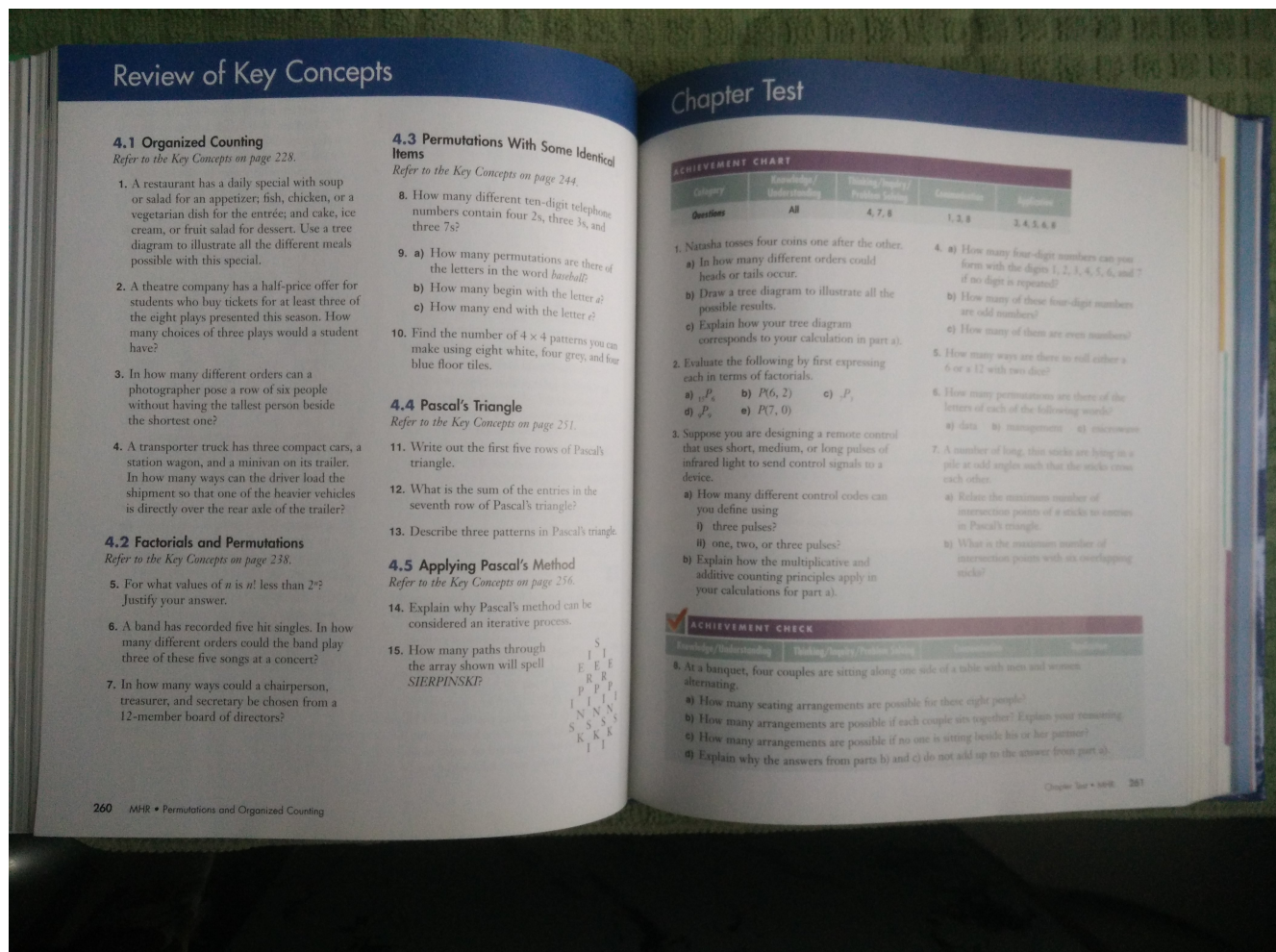
2. **Page 261:** 1,3 and 8

3. **Page 294/295:** 10-22.

Review of Key Concepts

4.1 Organized Counting
Refer to the Key Concepts on page 228.

1. A restaurant has a daily special with soup or salad for an appetizer; fish, chicken, or a vegetarian dish for the entrée; and cake, ice cream, or fruit salad for dessert. Use a tree diagram to illustrate all the different meals possible with this special.

2. A theatre company has a half-price offer for students who buy tickets for at least three of the eight plays presented this season. How many choices of three plays would a student have?

3. In how many different orders can a photographer pose a row of six people without having the tallest person beside the shortest one?

4. A transporter truck has three compact cars, a station wagon, and a minivan on its trailer. In how many ways can the driver load the shipment so that one of the heavier vehicles is directly over the rear axle of the trailer?

4.2 Factorials and Permutations
Refer to the Key Concepts on page 238.

5. For what values of n is $n!$ less than 2^n? Justify your answer.

6. A band has recorded five hit singles. In how many different orders could the band play three of these five songs at a concert?

7. In how many ways could a chairperson, treasurer, and secretary be chosen from a 12-member board of directors?

4.3 Permutations With Some Identical Items
Refer to the Key Concepts on page 244.

8. How many different ten-digit telephone numbers contain four 2s, three 3s, and three 7s?

9. a) How many permutations are there of the letters in the word *baseball*?
   b) How many begin with the letter *a*?
   c) How many end with the letter *e*?

10. Find the number of 4×4 patterns you can make using eight white, four grey, and four blue floor tiles.

4.4 Pascal's Triangle
Refer to the Key Concepts on page 251.

11. Write out the first five rows of Pascal's triangle.

12. What is the sum of the entries in the seventh row of Pascal's triangle?

13. Describe three patterns in Pascal's triangle.

4.5 Applying Pascal's Method
Refer to the Key Concepts on page 256.

14. Explain why Pascal's method can be considered an iterative process.

15. How many paths through the array shown will spell *SIERPINSKI*?

```
        S
      I   I
    E   E   E
  R   R   R   R
P   P   P   P   P
  I   I   I   I
    N   N   N
      S   S
        K   K
          I
```

Chapter Test

1. Natasha tosses four coins one after the other.
   a) In how many different orders could heads or tails occur.
   b) Draw a tree diagram to illustrate all the possible results.
   c) Explain how your tree diagram corresponds to your calculation in part a).

2. Evaluate the following by first expressing each in terms of factorials.
   a) $_{11}P_4$ b) $P(6, 2)$ c) $_5P_5$
   d) $_9P_2$ e) $P(7, 0)$

3. Suppose you are designing a remote control that uses short, medium, or long pulses of infrared light to send control signals to a device.
   a) How many different control codes can you define using
      i) three pulses?
      ii) one, two, or three pulses?
   b) Explain how the multiplicative and additive counting principles apply in your calculations for part a).

4. a) How many four-digit numbers can you form with the digits 1, 2, 3, 4, 5, 6, and 7 if no digit is repeated?
   b) How many of these four-digit numbers are odd numbers?
   c) How many of them are even numbers?

5. How many ways are there to roll either a 6 or a 12 with two dice?

6. How many permutations are there of the letters of each of the following words?
   a) data   b) management   c) microwave

7. A number of long, thin sticks are lying in a pile at odd angles such that the sticks cross each other.
   a) Relate the maximum number of intersection points of a sticks to entries in Pascal's triangle.
   b) What is the maximum number of intersection points with six overlapping sticks?

8. At a banquet, four couples are sitting along one side of a table with men and women alternating.
   a) How many seating arrangements are possible for these eight people?
   b) How many arrangements are possible if each couple sits together? Explain your reasoning.
   c) How many arrangements are possible if no one is sitting beside his or her partner?
   d) Explain why the answers from parts b) and c) do not add up to the answer from part a).

Apply, Solve, Communicate

B

5. Using the binomial theorem and patterns in Pascal's triangle, simplify each of the following.

a) $_9C_0 + _9C_1 + \ldots + _9C_9$

b) $_{12}C_0 - _{12}C_1 + _{12}C_2 - \ldots - _{12}C_{11} + _{12}C_{12}$

c) $\sum_{r=0}^{13} {}_{13}C_r$

d) $\sum_{r=0}^{n} {}_nC_r$

6. If $\sum_{r=0}^{n} {}_nC_r = 16\,384$, determine the value of n.

7. a) Write formulas in combinatorial form for the following. (Refer to section 4.4, if necessary.)

i) the sum of the squares of the terms in the nth row of Pascal's triangle

ii) the result of alternately adding and subtracting the squares of the terms in the nth row of Pascal's triangle

iii) the number of diagonals in an n-sided polygon

b) Use your formulas from part a) to determine

i) the sum of the squares of the terms in row 15 of Pascal's triangle

ii) the result of alternately adding and subtracting the squares of the terms in row 12 of Pascal's triangle

iii) the number of diagonals in a 14-sided polygon

8. How many terms would be in the expansion of $(x^2 + z)^9$?

9. Use the binomial theorem to expand and simplify the following.

a) $(x+y)^7$ b) $(2x+3y)^4$

c) $(2x-5y)^5$ d) $(x^2+5)^4$

e) $(3a^2+4c)^5$ f) $5(2p-6z^2)^5$

10. Communication

a) Find and simplify the first five terms of the expansion of $(3x + y)^{10}$.

b) Find and simplify the first five terms of the expansion of $(3x - y)^{10}$.

c) Describe any similarities and differences between the terms in parts a) and b).

11. Use the binomial theorem to expand and simplify the following.

a) $\left(x^2 - \dfrac{1}{x}\right)^5$ b) $\left(2y + \dfrac{3}{y^2}\right)^4$

c) $(\sqrt{x} + 2x^2)^6$ d) $\left(k + \dfrac{k}{m^2}\right)^5$

e) $\left(\sqrt{y} - \dfrac{2}{\sqrt{y}}\right)^7$ f) $2\left(3m^2 - \dfrac{2}{\sqrt{m}}\right)^4$

12. Application Rewrite the following expansions in the form $(a + b)^n$, where n is a positive integer.

a) $x^6 + 6x^5y + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6xy^5 + y^6$

b) $y^{12} + 8y^9 + 24y^6 + 32y^3 + 16$

c) $243a^5 - 405a^4b + 270a^3b^2 - 90a^2b^3 + 15ab^4 - b^5$

13. Communication Use the binomial theorem to simplify each of the following. Explain your results.

a) $\left(\dfrac{1}{2}\right)^5 + 5\left(\dfrac{1}{2}\right)^5 + 10\left(\dfrac{1}{2}\right)^5 + 10\left(\dfrac{1}{2}\right)^5 + 5\left(\dfrac{1}{2}\right)^5 + \left(\dfrac{1}{2}\right)^5$

b) $(0.7)^7 + 7(0.7)^6(0.3) + 21(0.7)^5(0.3)^2 + \ldots + (0.3)^7$

c) $7^9 - 9 \times 7^8 + 36 \times 7^7 - \ldots - 7^9$

14. a) Expand $\left(x + \dfrac{2}{x}\right)^4$ and compare it with the expansion of $\dfrac{1}{x^4}(x^2 + 2)^4$.

b) Explain your results.

15. Use your knowledge of algebra and the binomial theorem to expand and simplify each of the following.

a) $(25x^2 + 30xy + 9y^2)^3$

b) $(3x - 2y)^5(3x + 2y)^5$

16. Application

a) Calculate an approximation for $(1.2)^9$ by expanding $(1 + 0.2)^9$.

b) How many terms do you have to evaluate to get an approximation accurate to two decimal places?

17. In a trivia contest, Adam has drawn a topic he knows nothing about, so he makes random guesses for the ten true/false questions. Use the binomial theorem to help find

a) the number of ways that Adam can answer the test using exactly four *true*s

b) the number of ways that Adam can answer the test using at least one *true*

✓ ACHIEVEMENT CHECK

Knowledge/ Understanding	Thinking/Inquiry/ Problem Solving	Communication	Application

18. a) Expand $(h + t)^5$.

b) Explain how this expansion can be used to determine the number of ways of getting exactly h heads when five coins are tossed.

c) How would your answer in part b) change if six coins are being tossed? How would it change for n coins? Explain.

C

19. Find the first three terms, ranked by degree of the terms, in each expansion.

a) $(x+3)(2x+5)^4$

b) $(2x+1)^5(4x-3)^5$

c) $(x^2-5)^6(x^3+2)^6$

20. Inquiry/Problem Solving

a) Use the binomial theorem to expand $(x + y + z)^2$ by first rewriting it as $[x + (y + z)]^2$.

b) Repeat part a) with $(x + y + z)^3$.

c) Using parts a) and b), predict the expansion of $(x + y + z)^4$. Verify your prediction by using the binomial theorem to expand $(x + y + z)^4$.

d) Write a formula for $(x + y + z)^n$.

e) Use your formula to expand and simplify $(x + y + z)^5$.

21. a) In the expansion of $(x + y)^5$, replace x and y with B and G, respectively. Expand and simplify.

b) Assume that a couple has an equal chance of having a boy or a girl. How would the expansion in part a) help find the number of ways of having k girls in a family with five children?

c) In how many ways could a family with five children have exactly three girls?

d) In how many ways could they have exactly four boys?

22. A simple code consists of a string of five symbols that represent different letters of the alphabet. Each symbol is either a dot (•) or a dash (–).

a) How many different letters are possible using this code?

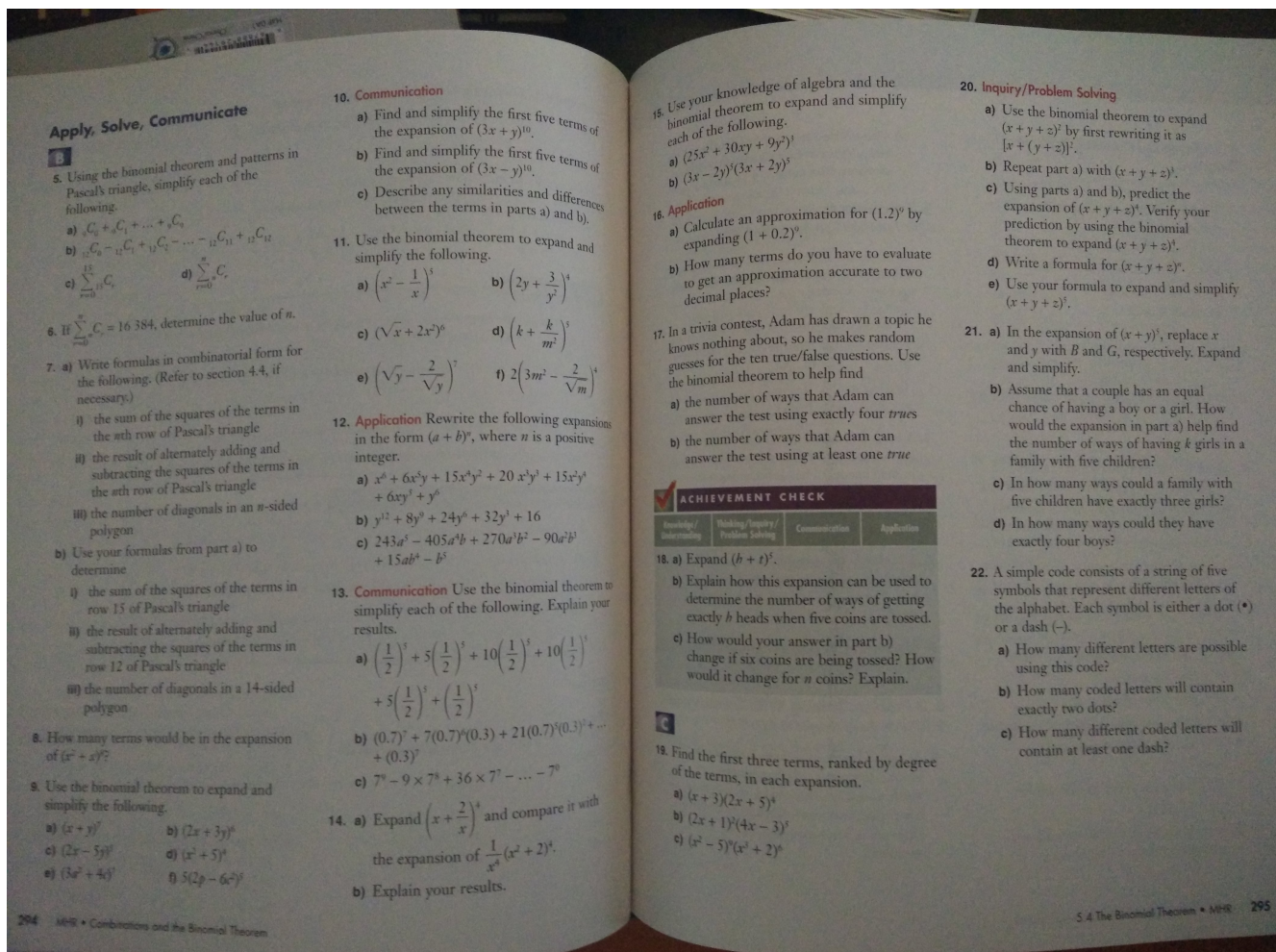b) How many coded letters will contain exactly two dots?

c) How many different coded letters will contain at least one dash?

Figure 2: Problems 10-22.